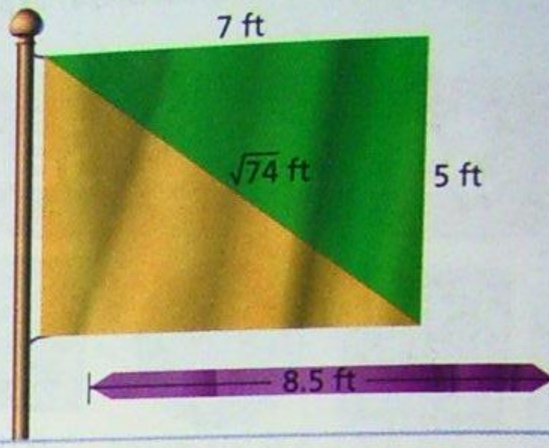
EXAMPLE 1

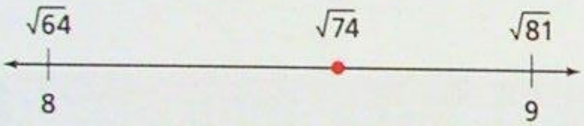
Darcy wants to add the ribbon shown along the diagonal of the rectangular flag she is designing. Does Darcy have enough ribbon? Explain.



Approximate $\sqrt{74}$ using perfect squares.

Because 74 lies between the two consecutive perfect squares 64 and 81, $\sqrt{74}$ is located between $\sqrt{64}$ and $\sqrt{81}$.

Because 74 is closer to 81 than 64, $\sqrt{74}$ is closer to $\sqrt{81}$, or 9.

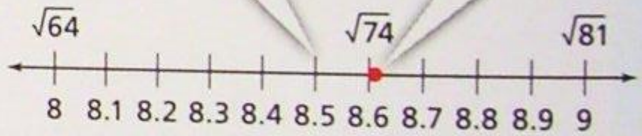


Find a better approximation by squaring decimals between 8 and 9. Then compare.

Reasoning Which decimals can you use to find a better approximation? MP.2

$8.5 \times 8.5 = 72.25$
This approximation is too low.

$8.6 \times 8.6 = 73.96$
This is a good approximation.



The length of the diagonal, $\sqrt{74}$, is about 8.6 feet. Darcy does not have enough ribbon.

Try It!

Between which two whole numbers is $\sqrt{12}$?

$\boxed{9} < 12 < \boxed{16}$

$\boxed{\sqrt{9}} < \sqrt{12} < \boxed{\sqrt{16}}$

closer to 3 → $\boxed{3} < \sqrt{12} < \boxed{4}$

Convince Me! Which of the two numbers is a better estimate for $\sqrt{12}$? Explain.

Since 9 is the closer # to 12, $\sqrt{12}$ is closer to $\sqrt{9} = 3$.

STEP 1 Approximate $\sqrt{32}$ by using perfect squares.

$25 < 32 < 36$

$\sqrt{25} < \sqrt{32} < \sqrt{36}$

$5 < \sqrt{32} < 6$

Then find a better approximation by using decimals.

$5.5 \times 5.5 = 30.25$ $\quad$ $5.6 \times 5.6 = 31.36$ $\quad$ $5.7 \times 5.7 = 32.49$

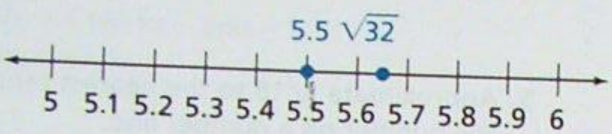
$5.6 < \sqrt{32} < 5.7$

Look for Relationships
To compare irrational numbers and locate them on a number line, you can use their rational approximations. MP.7

STEP 2 Approximate 5.51326... as a rational number by rounding to the nearest tenth.

$5.51326... \approx 5.5$

STEP 3 Plot each approximation on a number line to compare.



So, $5.51326... < \sqrt{32}$.

EXAMPLE 3 Compare and Order Rational and Irrational Numbers

Compare and order the numbers below.

$$\pi^2, 9\tfrac{1}{2}, 9.8, 9.\overline{5}, \sqrt{94}$$

STEP 1 Use rational approximation to estimate the values of irrational numbers.

$\pi^2 \approx 3.14 \times 3.14 \approx 9.8596$ (calculator)

$9\frac{1}{2} = 9.5$

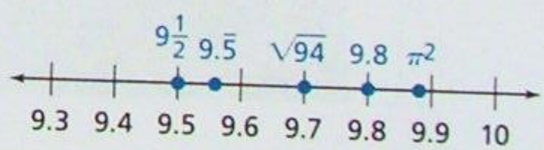
9.8

$9.\overline{5} = 9.5555...$

$\sqrt{94} \approx 9.7$

$\sqrt{81}$ $\sqrt{94}$ $\sqrt{100}$
9 10 closer to 10

STEP 2 Plot each approximation on a number line.



So, $9\frac{1}{2} < 9.\overline{5} < \sqrt{94} < 9.8 < \pi^2$.

Try It!

$\sqrt{9}$ $\sqrt{11}$ $\sqrt{16}$
closer to 3 3 4

change to the decimal approximations!

Compare and order the following numbers:

$\sqrt{11}, 2\frac{1}{4}, -2.5, 3.\overline{6}, -3.97621...$

3 2.25 $3.\overline{6}$

$\boxed{-3.97621, -2.5, 2\tfrac{1}{4}, \sqrt{11}, 3.\overline{6}}$

least greatest