[2.3] The Quadratic Formula

Standard Form: $y = ax^2 + bx + c$

$$x = \frac{-b \pm \sqrt{b^2 - (4ac)}}{2a}$$

① $x^2 + 3x - 10 = 0$

$a = 1 \quad b = 3 \quad c = -10$

always positive

$$x = \frac{-3 \pm \sqrt{9 - (4 \cdot 1 \cdot -10)}}{2 \cdot 1}$$

$$x = \frac{-3 \pm \sqrt{9 - (-40)}}{2}$$

$$x = \frac{-3 \pm \sqrt{49}}{2}$$

$$\frac{-3+7}{2} \qquad \frac{-3-7}{2}$$

$$= \frac{4}{2} = 2 \qquad = \frac{-10}{2} = -5$$

$\boxed{-5, 2}$

② $2x^2 - 7 = 5x$

$2x^2 - 5x - 7 = 0$

$a = 2 \quad b = -5 \quad c = -7$

$$x = \frac{5 \pm \sqrt{25 - -56}}{4}$$

$$x = \frac{5 \pm \sqrt{81}}{4}$$

$$\frac{5+9}{4} \qquad \frac{5-9}{4}$$

$$\frac{14}{4} = \frac{7}{2} \qquad \frac{-4}{4} = -1$$

$\boxed{-1, \frac{7}{2}}$

Factoring

$x^2 + 3x - 10 = 0$

$(x - 2)(x + 5) =$

$\boxed{2 \quad -5}$

Factoring

$2x^2 - 5x - 7 = 0$

$(2x - 7)(x + 1) = 0$

$\boxed{\frac{7}{2} \quad -1}$

③ $x^2 + 6 - 4x = 0$

$x^2 - 4x + 6 = 0$

$a = 1 \quad b = -4 \quad c = 6$

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 6}}{2}$$

$$x = \frac{4 \pm \sqrt{16 - 24}}{2}$$

$$x = \frac{4 \pm \sqrt{-8}}{2}$$ ← If negative...

No Real Solution

$\sqrt{-8} = i\sqrt{8}$

$2i\sqrt{2}$

④ $2x^2 - 4x + 1 = 0$

$a = 2 \quad b = -4 \quad c = 1$

$$x = \frac{4 \pm \sqrt{16 - 4 \cdot 2 \cdot 1}}{4}$$

$$x = \frac{4 \pm \sqrt{16 - 8}}{4}$$

$$x = \frac{4 \pm \sqrt{8}}{4}$$

$$x = \frac{\overset{2}{\cancel{4}} \pm \overset{1}{\cancel{2}}\sqrt{2}}{\cancel{4}2}$$

$$x = \boxed{\frac{2 \pm \sqrt{2}}{2}} \approx \boxed{1.7 \text{ ; } 0.3}$$

Approx.

$\frac{2 + 1.4}{2} \qquad \frac{2 - 1.4}{2}$

$\frac{3.4}{2} = 1.7 \qquad \frac{0.6}{2} = 0.3$

2–3 The Quadratic Formula

Objective: To learn the quadratic formula and use it to solve equations.

Complete Even #s only

The Quadratic Formula

The solutions of a quadratic equation in the form of $ax^2 + bx + c = 0$, $a \neq 0$ and $b^2 - 4ac \geq 0$ are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 1 Use the quadratic formula to solve $3x^2 + 5x - 2 = 0$.

Solution $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 3$, $b = 5$, and $c = -2$.

$x = \dfrac{-5 \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)}$ Substitute the given values of a, b, and c.

$= \dfrac{-5 \pm \sqrt{25 + 24}}{6}$

$= \dfrac{-5 \pm \sqrt{49}}{6} = \dfrac{-5 \pm 7}{6}$

$x = \dfrac{-5 + 7}{6} = \dfrac{2}{6} = \dfrac{1}{3}$ or $x = \dfrac{-5 - 7}{6} = \dfrac{-12}{6} = -2$

The check is left to you. The solution set is $\left\{\frac{1}{3}, -2\right\}$.

Use the quadratic formula to solve each equation.

1. $x^2 + 3x - 10 = 0$
2. $x^2 - 8x + 7 = 0$
3. $x^2 + 2x - 3 = 0$
4. $x^2 - 14x + 24 = 0$
5. $n^2 + 5n - 6 = 0$
6. $x^2 - 6x - 40 = 0$
7. $2x^2 + 3x - 2 = 0$
8. $3u^2 - 5u - 2 = 0$
9. $3x^2 - 10x - 8 = 0$
10. $3x^2 - 2x - 1 = 0$
11. $2x^2 - 5x - 7 = 0$
12. $5x^2 + 6x - 8 = 0$

Example 2 Use the quadratic formula to solve $x^2 = x - 6$.

Solution $x^2 - x + 6 = 0$ Rewrite the equation in standard form.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 1$, $b = -1$, and $c = 6$.

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(6)}}{2(1)} = \dfrac{1 \pm \sqrt{1 - 24}}{2} = \dfrac{1 \pm \sqrt{-23}}{2}$

Since $\sqrt{b^2 - 4ac} = \sqrt{-23}$ and $\sqrt{-23}$ isn't a real number, there is *no real solution*.

Use the quadratic formula to solve each equation.

13. $x^2 - 4x + 6 = 0$
14. $2x^2 = 3x - 1$
15. $x^2 - 4x = 30$
16. $2x^2 + 2x + 5 = 0$
17. $4x^2 + 20x = -9$
18. $3x^2 - 3x + 4 = 0$

Example 3 Use the quadratic formula to solve $2x^2 - 3x - 4 = 0$. Give irrational roots in simplest radical form and then approximate them to the nearest tenth. You may wish to use a calculator.

Solution

$2x^2 - 3x - 4 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 2$, $b = -3$, and $c = -4$.

$x = \dfrac{3 \pm \sqrt{9 - 4(2)(-4)}}{2(2)}$ Substitute the given values of a, b, and c.

$= \dfrac{3 \pm \sqrt{9 + 32}}{4}$ Simplify.

$= \dfrac{3 \pm \sqrt{41}}{4}$

Since $\sqrt{41} \approx 6.40$, $x \approx \dfrac{3 + 6.4}{4} = 2.35 \approx 2.4$

or $x \approx \dfrac{3 - 6.4}{4} = -0.85 \approx -0.9$

The check is left to you.

The solution set is $\left\{\dfrac{3 + \sqrt{41}}{4}, \dfrac{3 - \sqrt{41}}{4}\right\}$ or $\{2.4, -0.9\}$.

Use the quadratic formula to solve each equation. Give irrational roots in simplest radical form and then approximate them to the nearest tenth. You may wish to use a calculator.

19. $2x^2 = 8x - 5$
20. $3x^2 + 2x = 2$
21. $x^2 - 4x - 10 = 0$
22. $x^2 - 4x - 2 = 0$
23. $2x^2 - 4x + 1 = 0$
24. $3x^2 - 8x + 2 = 0$
25. $2x^2 + 1 = 3x$
26. $3x^2 + x = 2$
27. $4x^2 - 11x = 3$