

Solving Equations by Factoring

IF $a \cdot b = 0$, then
 $a = 0$ and/or $b = 0$.

$$\textcircled{1} \quad \underbrace{(y+4)}_{\substack{y+4=0 \\ y=-4}} \underbrace{(y-5)}_{\substack{y-5=0 \\ y=5}} = 0$$

$$\boxed{\{-4, 5\}}$$

$$\textcircled{2} \quad x \underbrace{(x-10)}_{\substack{x-10=0 \\ x=10}} \underbrace{(x+10)}_{\substack{x+10=0 \\ x=-10}} = 0$$

$$\boxed{\{-10, 0, 10\}}$$

$$\textcircled{3} \quad \underbrace{3x}_{\substack{3x=0 \\ x=0}} \underbrace{(2x-1)}_{\substack{2x-1=0 \\ 2x=1 \\ x=\frac{1}{2}}} \underbrace{(3x+2)}_{\substack{3x+2=0 \\ 3x=-2 \\ x=-\frac{2}{3}}} = 0$$

$$\boxed{\{-\frac{2}{3}, 0, \frac{1}{2}\}}$$

$$\textcircled{4} \quad 2x^2 - 2x - 24 = 0 \quad \leftarrow \text{Has to be zero}$$
$$2 \left\{ \begin{array}{l} x^2 - x - 12 \\ (x+3)(x-4) \end{array} \right\} = 0$$
$$\underbrace{(x+3)}_{\substack{x+3=0 \\ x=-3}} \underbrace{(x-4)}_{\substack{x-4=0 \\ x=4}} = 0$$

$$\boxed{\{-3, 4\}}$$

$$\textcircled{5} \quad x^2 - 12x + 27 = 0$$
$$(x-9)(x-3) = 0$$
$$x=9 \quad x=3$$

$$\boxed{\{3, 9\}}$$

$$\textcircled{6} \quad 3x^2 + 1 = 4x$$
$$3x^2 - 4x + 1 = 0$$
$$(3x-1)(x-1) = 0$$
$$\substack{3x=1 \\ x=\frac{1}{3}} \quad \substack{x=1$$

$$\boxed{\{\frac{1}{3}, 1\}}$$

$$\textcircled{7} \quad -6x = -x^2 - 9$$

$$x^2 - 6x + 9 = 0$$

$$(x - 3)(x - 3) = 0$$

$$x = 3$$

$\boxed{\{3\}}$

Solving Equations by Factoring

Example 1 Solve $(x - 1)(x + 3) = 0$.

Solution Since the product of factors is 0, one of the factors on the left side must equal 0.

$$\begin{array}{l} x - 1 = 0 \quad \text{or} \quad x + 3 = 0 \\ x = 1 \qquad \qquad \qquad x = -3 \end{array}$$

The solution set is $\{1, -3\}$. Just by looking at the original equation, you can see that when $x = 1$ or $x = -3$, the product will be 0.

Example 2 Solve $3n(n - 2)(n - 5) = 0$.

Solution $3n = 0$ or $n - 2 = 0$ or $n - 5 = 0$
 $n = 0$ $n = 2$ $n = 5$ The solution set is $\{0, 2, 5\}$.

CAUTION Never transform an equation by dividing by an expression containing a variable. Notice that in Example 2, the solution 0 would have been lost if both sides of the equation had been divided by $3n$.

Solve.

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|----------------------------|-------------------------|----------------------------|
| 1. $(y + 4)(y - 5) = 0$ | 2. $0 = (n + 1)(n + 8)$ | 3. $10n(n - 2) = 0$ |
| 4. $2x(x - 10) = 0$ | 5. $(p - 1)(p - 7) = 0$ | 6. $0 = 2n(n - 1)(n - 3)$ |
| 7. $x(2x - 1)(2x + 1) = 0$ | 8. $0 = n(n - 6)$ | 9. $0 = 3x(4x - 1)(x - 2)$ |

Example 3 Solve the quadratic equation $2x^2 - x = 3$.

Solution

| | |
|---|-----------------------------|
| 1. Transform the equation into standard form. | $2x^2 - x - 3 = 0$ |
| 2. Factor the left side. | $(2x - 3)(x + 1) = 0$ |
| 3. Set each factor equal to 0 and solve. | $2x - 3 = 0$ or $x + 1 = 0$ |
| | $2x = 3$ $x = -1$ |
| | $x = \frac{3}{2}$ |

Solve.

- | | | |
|--------------------------|--------------------------|--------------------------|
| 10. $x^2 - x - 12 = 0$ | 11. $x^2 - 12x + 27 = 0$ | 12. $0 = x^2 - 4x - 32$ |
| 13. $0 = m^2 + 3m - 54$ | 14. $x^2 - 4x + 3 = 0$ | 15. $x^2 - 10x - 24 = 0$ |
| 16. $0 = n^2 - n$ | 17. $y^2 = 12y$ | 18. $6k^2 = 2k$ |
| 19. $x^2 + 16 = 8x$ | 20. $a^2 = 10 - 3a$ | 21. $3x^2 - x = 2$ |
| 22. $0 = x^2 + 12x + 35$ | 23. $y^2 + 5y = 14$ | 24. $x^2 = 5x + 36$ |
| 25. $4m^2 - 25 = 0$ | 26. $r^2 + 8 = 9r$ | 27. $6n^2 - n = 2$ |
| 28. $3x^2 + 1 = 4x$ | 29. $3a^2 = 6a$ | 30. $3p^2 - 14p = 80$ |
| 31. $2x^2 = 10 + x$ | 32. $3p^2 + 17p = -10$ | 33. $3x^2 + 1 = 4x$ |