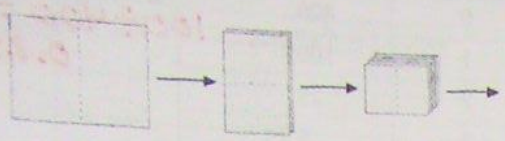
...tial Decay...Introduction

Size of the Ballots

In Problem 1.1, you read about ballots Chen, the secretary of the SGA, is making for a meeting. Recall that Chen cuts a sheet of paper in half, stacks the two pieces and cuts them in half, stacks the resulting four pieces and cuts then in half, and so on.



You investigated the pattern in the number of ballots created by each cut. In this problem, you will look at the pattern in the areas of the ballots.

A. The paper Chen starts with has an area of 64 square inches. Complete the table to show the area of a ballot after each of the first 10 cuts.

#of Cuts	Area (in^2)
0	64
1	32
2	16
3	8
4	4
5	2
6	1
7	$\frac{1}{2}$
8	$\frac{1}{4}$
9	$\frac{1}{8}$
10	$\frac{1}{16}$

$\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$ $\cdot\frac{1}{2}$

B. How does the area of a ballot change with each cut?

Multiply $\frac{1}{2}$

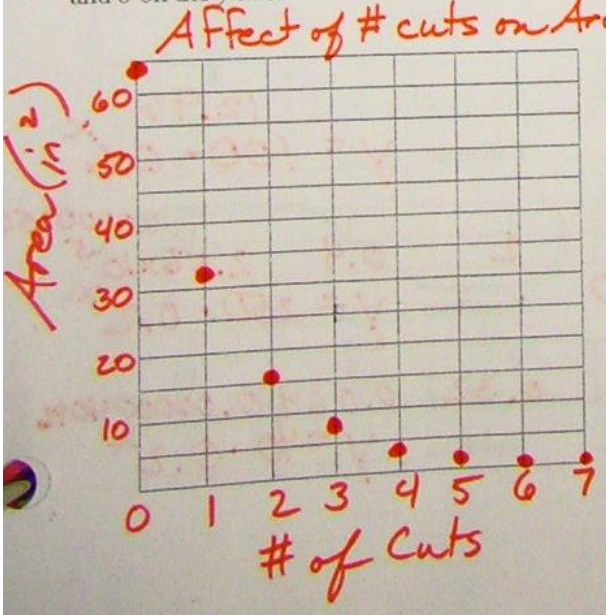
C. Write an equation for the area, A, of a ballot after any cut, n.

$A = (\frac{1}{2})^n \cdot 64$

D. How is the pattern of change in the area different from the exponential growth patterns you studied? How is it similar?

Growth & decay have a pattern of multiplying by the same #.

E. Make a graph of the first 7 cuts. Use an interval of 1 on the x-axis and 5 on the y-axis.

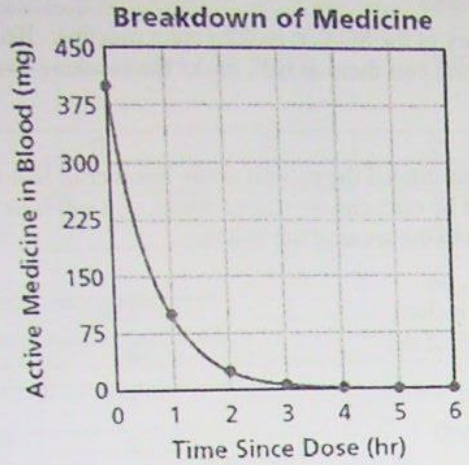


discrete data (don't connect)

Exponential patterns like this, in which a quantity decreases at each stage, show **exponential decay**. The factor the quantity is multiplied by at each stage is called a **decay factor**. A decay factor is always less than 1, but greater than zero. In this ballot problem, the decay factor is $\frac{1}{2}$.

Fighting Fleas

fter an animal receives a preventive flea medicine, the medicine breaks down in the animal's bloodstream. Vith each hour, there is less medicine in the blood. The table and graph show the amount of medicine in a og's bloodstream each hour for 6 hours after receiving a 400-milligram dose.



Breakdown of Medicine

Time Since Dose (hr)	Active Medicine in Blood (mg)
0	400
1	100
2	25
3	6.25
4	1.5625
5	0.3907
6	0.0977

100÷400 = 0.25

Study the pattern of change in the graph and the table.

1. How does the amount of active medicine in the dog's blood change from one hour to the next?

Divide by 4 or Multiply by $\frac{1}{4}$

2. What is the decay factor? (Show your work.) What is the original amount?

100÷400=0.25 400

3. Write an equation to model the relationship between the number of hours, h, since the dose is given and the milligrams of active medicine, m.

$m = 400 \cdot (0.25)^h$

4. How is the graph for this problem similar to the graph you made in the "Ballot Area" problem?

The graph decreases & both are ramps.

) Tables to Equations Complete each table. State the decay factor, the original # (y-intercept), and write ıe equation.

x	0	1	2	3	4
y	100	60	36	21.6	12.96

Decay factor: 60÷100= 0.6 Original #: 100 Equation: $y = 100 \cdot 0.6^x$

x	0	1	2	3	4	10
y	250	50	10	2	0.4	2.56×10^{-5}

0.0000256=

Decay factor: 50÷250 =0.2 Original #: 250 Equation: $y = 250 \cdot 0.2^x$

x	0	1	2	3	4	10
y	40	8	1.6	0.32	0.064	0.00000409

Decay factor: 8÷40 = 0.2 Original #: 40 Equation: $y = 40 \cdot 0.2^x$

2. Penicillin decays exponentially in the human body. Suppose you receive a 300-mg dose of penicillin to combat strep throat. About 180 mg will remain active in your blood after 1 day.

a. Assume the amount of penicillin active in your blood decreases exponentially. Make a table showing the amount of active penicillin in your blood for 7 days after a 300 mg dose.

Time Since Dose(days)	Active Medicine in Blood (mg)
0	
1	
2	
3	
4	
5	
6	
7	

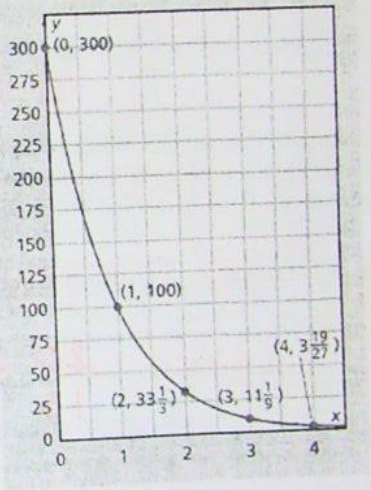
b. Write an equation for the relationship between the number of days, d, since you took the penicillin and the amount of medicine, m, remaining active in your blood.

_____

c. What would be the equation if you had taken a 400-mg dose?

_____

3. The graph toward the right shows an exponential decay relationship.

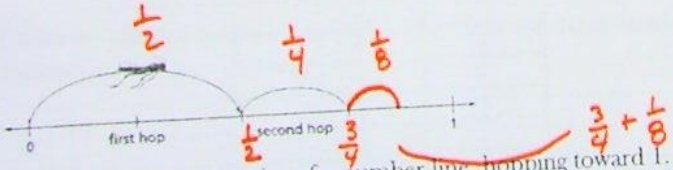


a. What is the decay factor? _____

b. What is the y-intercept? _____

c. Write the equation for the graph.

_____



4. A cricket is on the 0 point of a number line, hopping toward 1. She covers half the distance form her current location to 1 with each hop. So, she will be at $\frac{1}{2}$ after one hop, $\frac{3}{4}$ after two hops, and so on.

a. Make a table showing the cricket's location for the first 10 hops.

Hop	0	1	2	3	4	5	6	7	8	9	10
Location											

b. Where will the cricket be after n hops? _____

c. Will the cricket ever get to 1? Explain. _____

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