Beaumont Middle School
8th Grade, 2017-2018
Advanced Math8

Name: ______________________

Transformations

Reflections

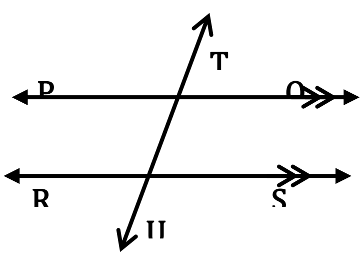
Rotations

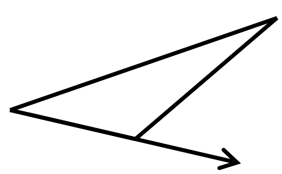
Dilations

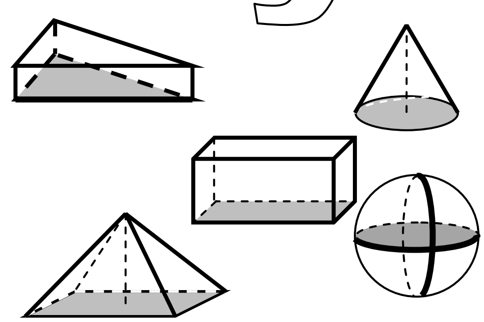
Translations

- I can recognize and create **reflections** on a coordinate grid.
- I can recognize and create **translations** on a coordinate grid.
- I can recognize and create **rotations** on a coordinate grid.
- I can identify congruency within reflections, translations, and rotations.
- I can recognize and create **dilation**s on a coordinate grid.
- I can identify similarity within dilations.
- I can write a general rule for transformations on a coordinate grid.
- I can recognize and create combined transformations.

Geometry





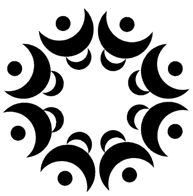


- I can determine an interior angle of a triangle given an interior and exterior angle measurement.
- I can use the angle relationships involving parallel lines and transversals to determine the measures of corresponding angles, alternate interior angles, alternate exterior angles.
- I can apply the volume formulas of right prisms, cylinders, pyramids, cones, and spheres.
- I can apply the formulas for volume to real-world and mathematical problems.

Reflectional Symmetry

An image has **Reflectional Symmetry** if there is at least one line which splits the image in half so that one side is the mirror image of the other. Reflectional symmetry is also called **line symmetry** or **mirror symmetry** because there is a line in the figure where a mirror could be placed, and the figure would look the same.

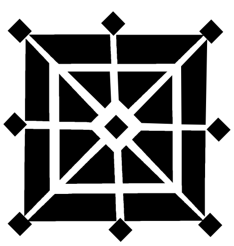
Think of a figure on a piece of paper, then folding the paper in two so that the two halves match up, or actually placing a mirror on the line of symmetry.



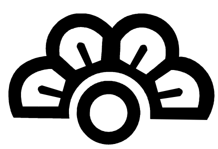
It is possible to have more than one line of reflectional symmetry.

Draw all of the **lines of symmetry** for each figure. Indicate the number of lines of symmetry for each figure. If the figure does not have reflectional symmetry, write "none."

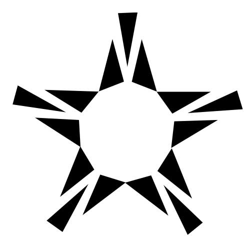
1.
#of lines:
_____



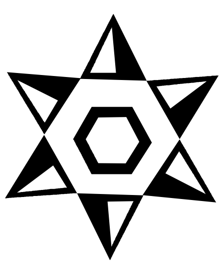
2.
#of lines:
_____



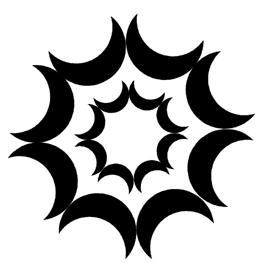
3.
#of lines:
_____



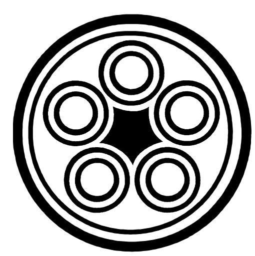
4.
#of lines:
_____



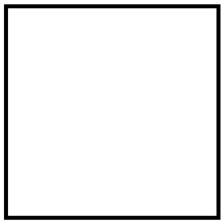
5.
#of lines:
_____



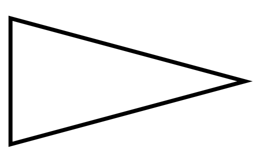
6.
#of lines:
_____



7.
#of lines:
_____



8.
#of lines:
_____



9.
#of lines:
_____

Notes for Reflectional Symmetry on a Coordinate Grid

The vertices of a polygon are listed. Graph and label each polygon and its image after a reflection over the given line. Name the coordinates of the image. State the rule for the transformation.

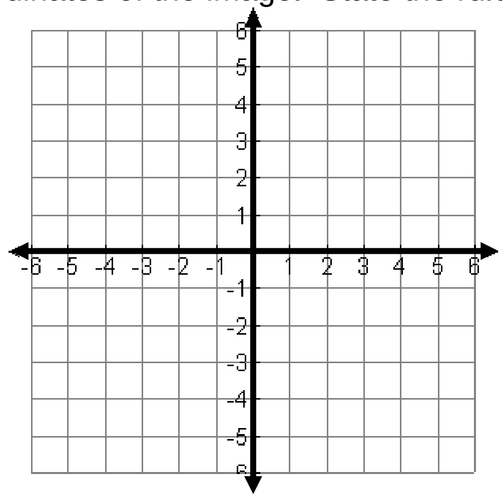
1. Reflect over the **x-axis**. (The x-axis can also be described as the line $y = 0$.)

   A (0, 2) → A' _______

   B (-2, 5) → B' _______

   C (-5, 1) → C' _______

General rule: ___________________



2. Reflect over the **y-axis**. (The y-axis can also be described as the line $x = 0$.)

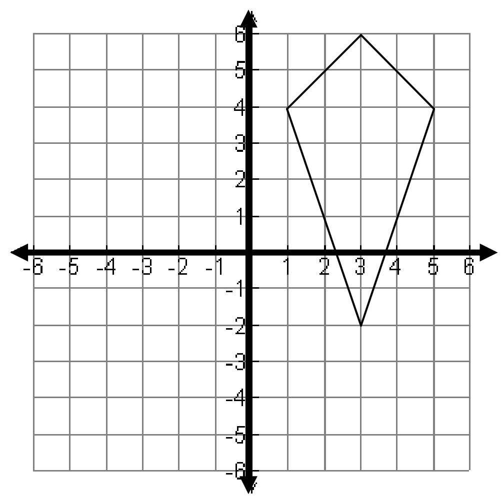
   E (1, 4) → E' _______

   F (3, -2) → F' _______

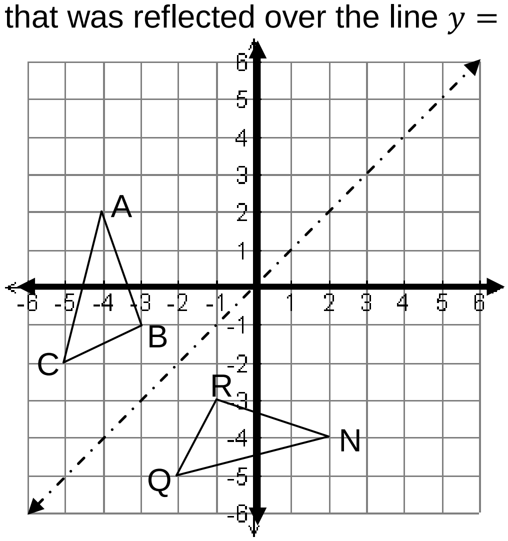
   G (5, 4) → G' _______

   H (3, 6) → H' _______

General rule: ___________________



A reflectional transformation results in a congruent figure. All angles and segments maintain the same measurements. Identify the congruent parts for the following triangle that was reflected over the line $y = x$.



$\overline{AB} \cong$ ______ $\quad \angle A \cong$ _______

$\overline{BC} \cong$ ______ $\quad \angle B \cong$ _______

$\overline{CA} \cong$ ______ $\quad \angle C \cong$ _______

$\Delta ABC \cong$ _______

State the coordinates of A and its corresponding vertex:

A: ____________ _______________

Write the general rule for a reflection over the line $x = y$

____

Homework for Reflectional Symmetry

The vertices of a polygon are listed. Graph and label each polygon and its image after a reflection over the given line. Name the coordinates of the image. State the rule for the transformation.

1. Reflect over the **y-axis**.

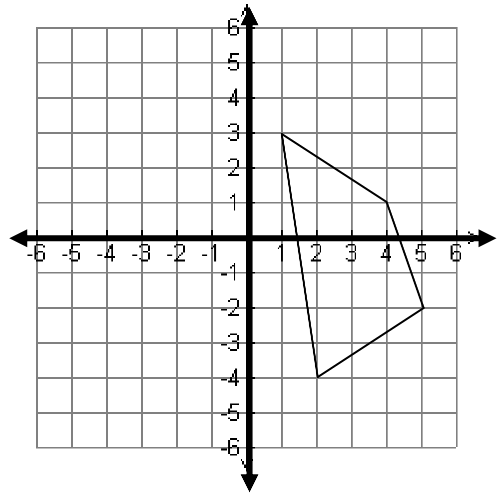
W (2,-4) → W' ________

X (1, 3) → X' ________

Y (4, 1) → Y' ________

Z (5, -2) → Z' ________

General rule: ____________________



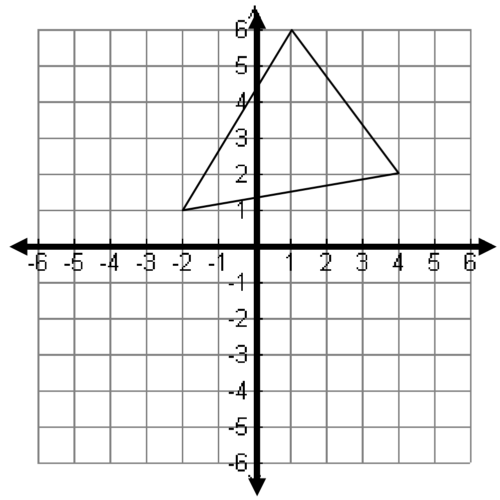
2. Reflect over the **x-axis**.

J (-2, 1) → J' ________

K (1, 6) → K' ________

L (4, 2) → L' ________

General rule: ____________________



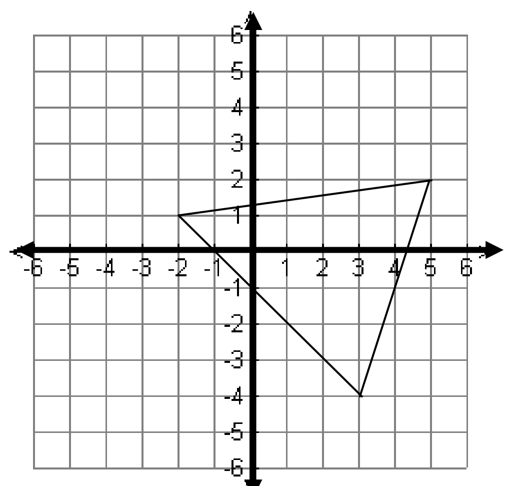
3. Reflect over the **x-axis**.
(The transformed polygon will overlap with the original.)

D (-2, 1) → D' ________

E (5, 2) → E' ________

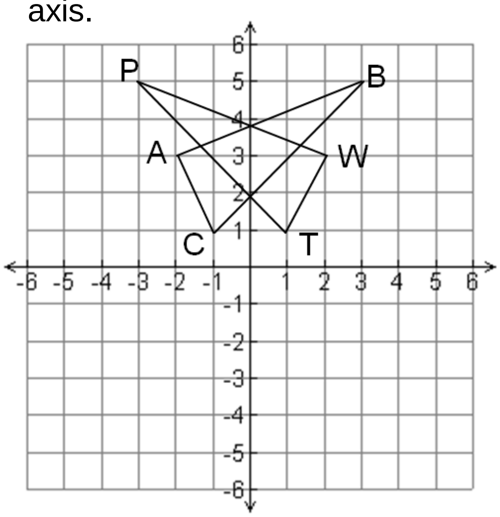
F (3, -4) → F' ________

General rule: ____________________



Homework is continued on the next page.

4. Identify the congruent parts for the following triangle that were reflected over the y-axis.



$\overline{AB} \cong$ ______

$\overline{BC} \cong$ ______

$\overline{CA} \cong$ ______

$\Delta ABC \cong$ _______

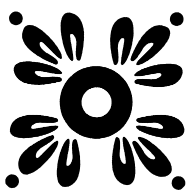
$\angle A \cong$ _______

$\angle B \cong$ _______

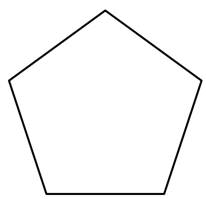
$\angle C \cong$ _______

Draw all of the **lines of symmetry** for each figure. If the figure does not have reflectional symmetry, write "none."

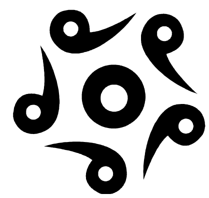
5.
#of lines: ______



6.
#of lines: ______

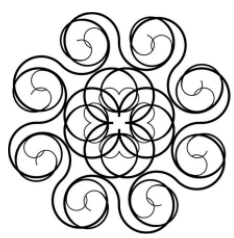


7.
#of lines: ______

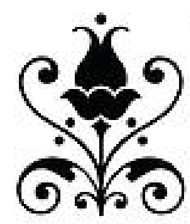


8.
#of lines: ______

9.
#of lines: ______



10.
#of lines: ______



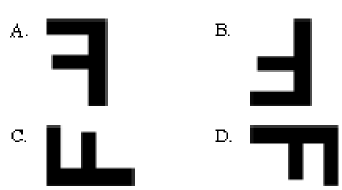
Multiple choice: The following are multiple choice questions. Circle the letter next to the answer.

11. 

Which of the following shows the image above reflected over the dotted line?

12. Use the letter in the box to answer the following question.

Which shows the letter after it has been FLIPPED ONCE?

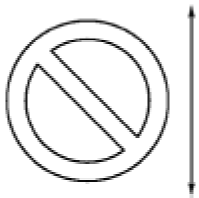


Homework is continued on the next page.

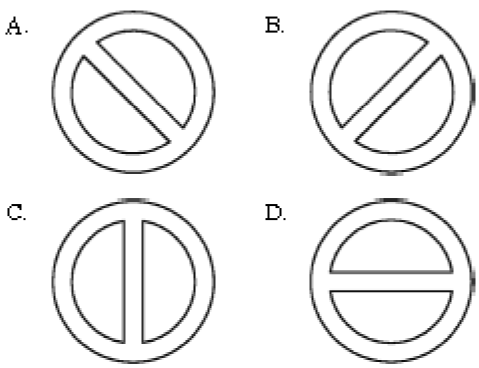
Multiple choice: The following are multiple choice questions. Circle the letter next to the answer.

13.

Anna noticed the following sign on the wall

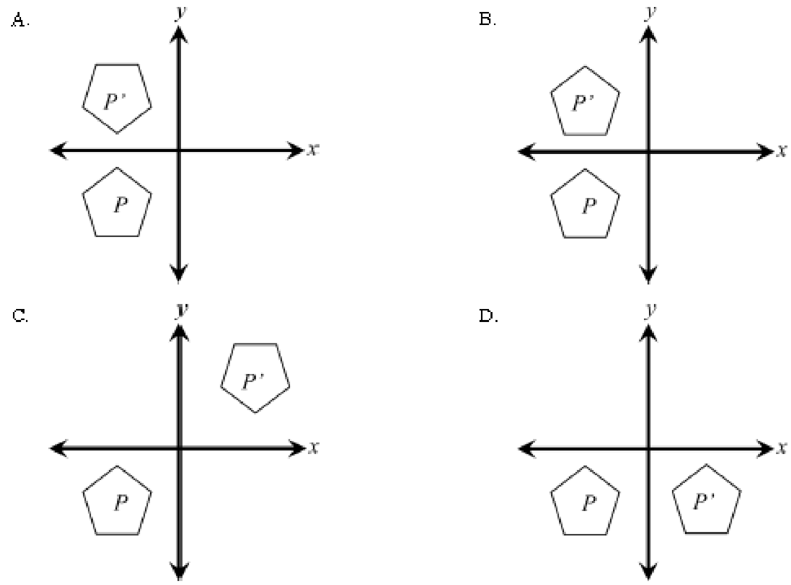


Which shows the sign after it has been flipped across the line?



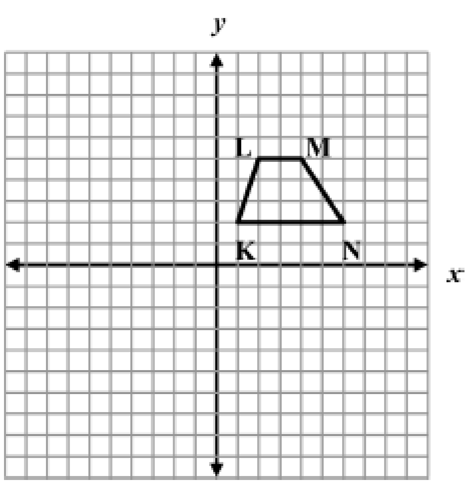
14.

Which figure is a reflection of figure P in respect to the x-axis?



15.

If trapezoid KLMN shown below is reflected across the x-axis to form trapezoid K'L'M'N', what are the apparent coordinates of M'?

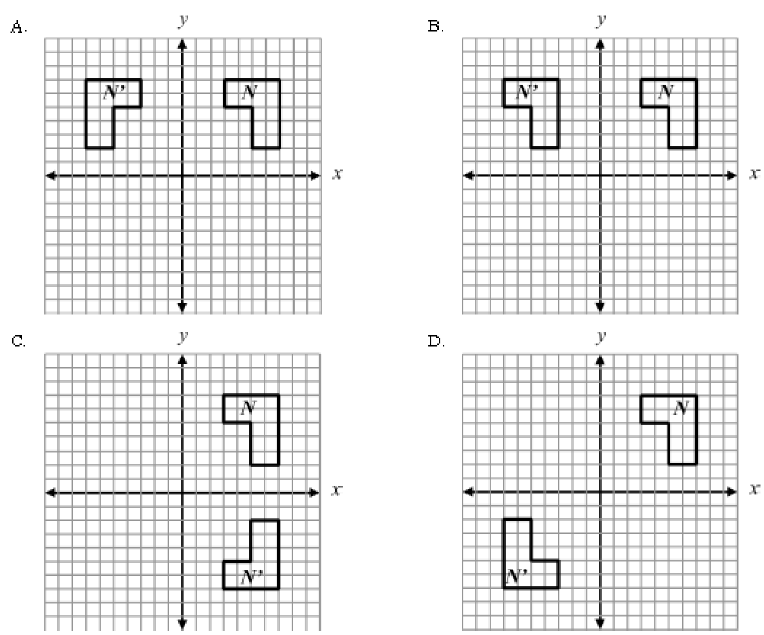


A. $(-4, 5)$ B. $(-4, -5)$

C. $(4, -5)$ D. $(4, 5)$

16.

Which of the following is a single reflection of figure N over the y-axis to form N'?

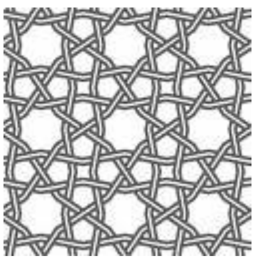


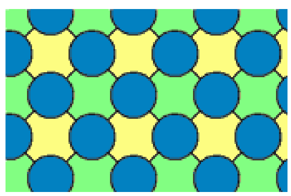
Translational Symmetry

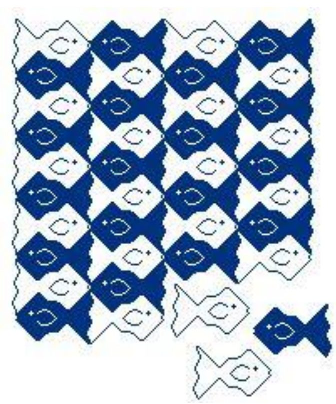
An image has **Translational Symmetry** if it can be divided by straight lines into a sequence of identical figures. Translational symmetry results from moving a figure a certain distance in a certain direction also called translating (moving) by a vector (length and direction).

A tessellation is created when a shape is repeated over and over again covering a plane without any gaps or overlaps.

Another word for a tessellation is a **tiling**.

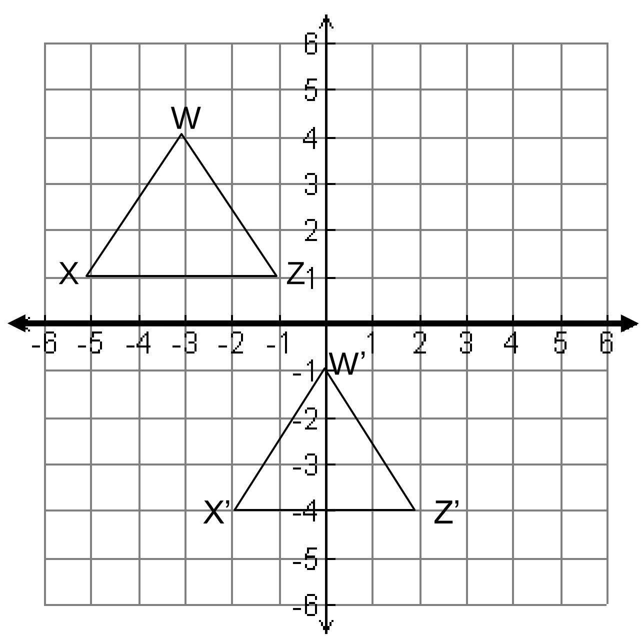






NOTES for Translational Symmetry

1. Name the coordinates of the image and its translation. State the rule for the transformation.



W _______ → W' _______

X _______ → X' _______

Z _______ → Z' _______

General rule: ____________________

NOTES for Translational Symmetry, continued

2. The vertices of a polygon are listed. Name the coordinates of the image's translation given the general rule for the transformation. Graph and label the original polygon and its image.

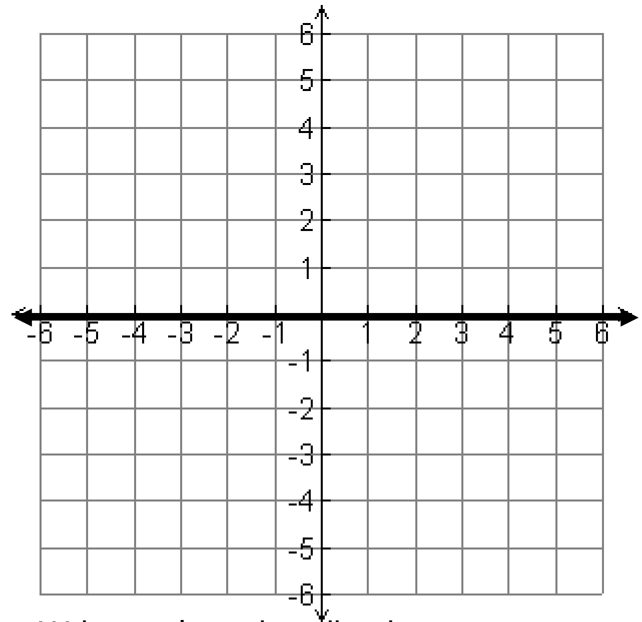
General rule: (x, y) → (x – 5, y + 2)

D (4, 2) → D' ________

E (6, -3) → E' ________

F (0, -3) → F' ________

G (2, 2) → G' ________

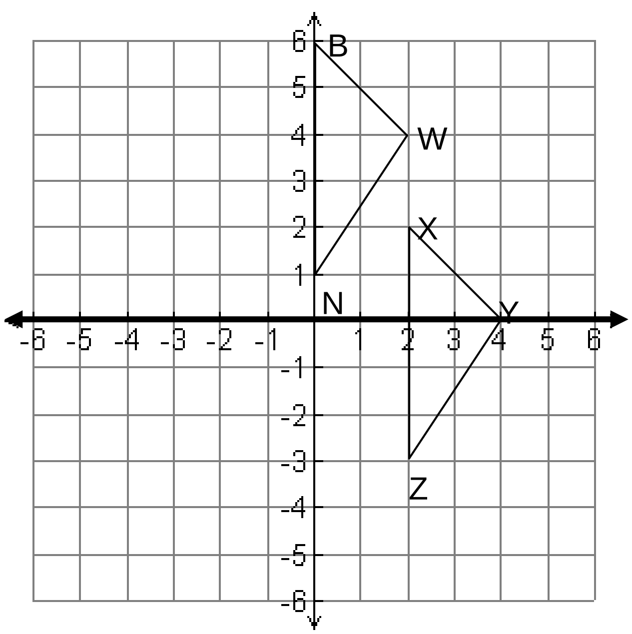


3. A point and its image after a translation are given. Write a rule to describe the translation.

a. The translation that takes A(8, -6) to A'(9, -3) (x, y) → ___________________

b. The translation that takes B(2, -10) to B'(2, -12) (x, y) → ___________________

4. A translational transformation also results in a congruent figure. Identify the congruent parts for triangle XYZ that was translated 2 units to the left and 4 units up.



$\overline{XY} \cong$ ______ $\angle W \cong$ _______

$\overline{YZ} \cong$ ______ $\angle N \cong$ _______

$\overline{ZX} \cong$ ______ $\angle B \cong$ _______

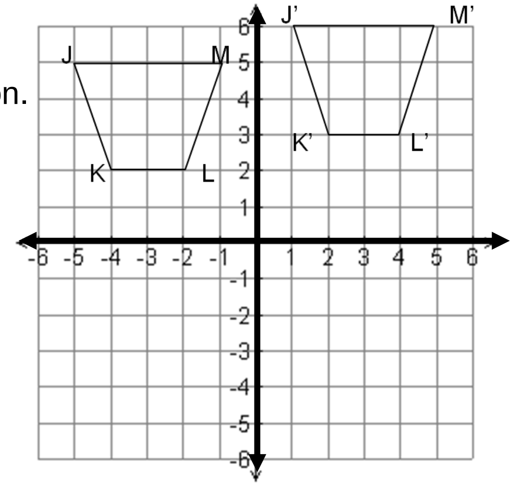
$\Delta XYZ \cong$ _______

State the coordinates of W and its corresponding vertex:

W: _____________ ________________

Write the general rule for the translation

____



HOMEWORK for Translational Symmetry

1. Name the coordinates of the image and its translation. State the rule for the transformation.

J _______ → J' _______

K _______ → K' _______

L _______ → L' _______

M _______ → M' _______

General rule: ____________________

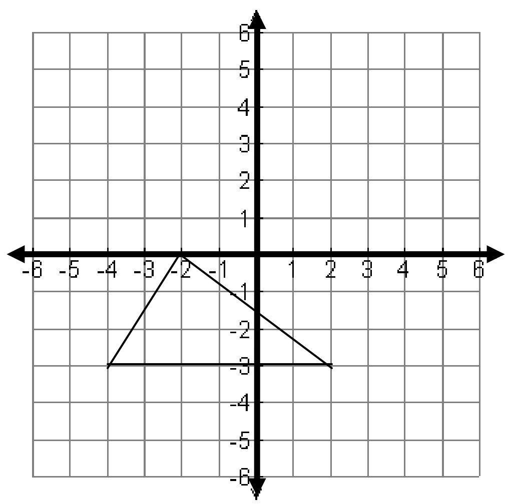
2. The vertices of a polygon are listed. Name the coordinates of the image's translation given the general rule for the transformation. Graph and label the original polygon and its image.

General rule: (x, y) → (x + 4, y – 2)

D (-2, 0) → D' _______

E (2, -3) → E' _______

F (-4, -3) → F' _______



3. The vertices of a polygon are listed. Name the coordinates of the image's translation given the general rule for the transformation. Graph and label the original polygon and its image.

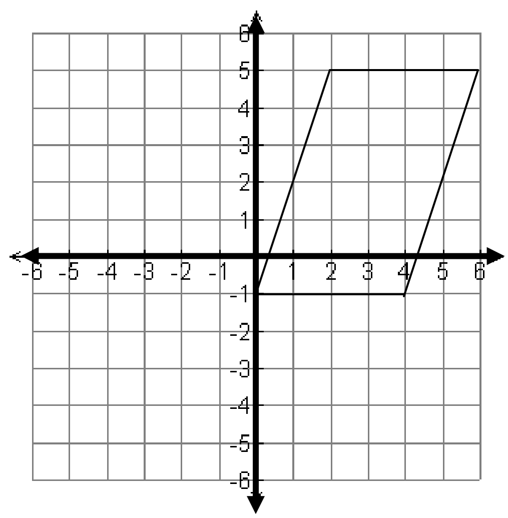
General rule: (x, y) → (x – 6, y)

R (6, 5) → R' _______

S (4, -1) → S' _______

T (0, -1) → T' _______

U (2, 5) → U' _______



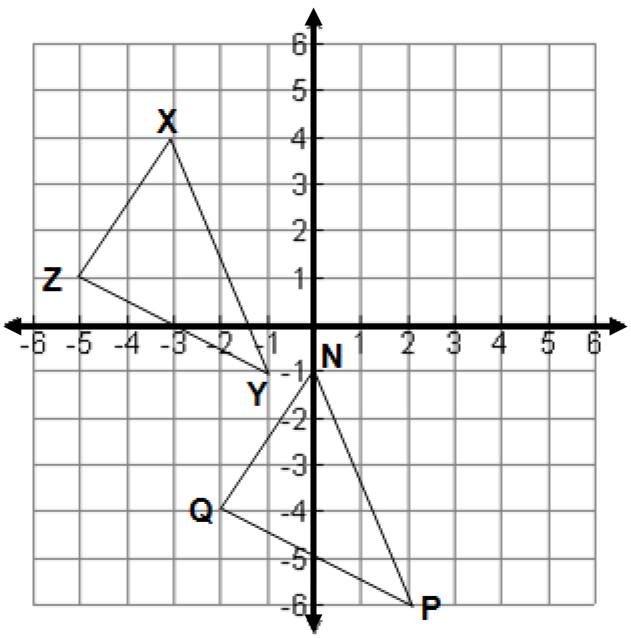
Homework is continued on the next page.

4. A point and its image after a translation are given. Write a rule to describe the translation.

a. The translation that takes A$(10, -5)$ to A'$(-5, -3)$ (x, y) → ________________

b. The translation that takes B$(2, -3)$ to B'$(7, -8)$ (x, y) → ________________

5. Identify the congruent parts for triangle XYZ that was translated 3 units to the right and 5 units down.



$\overline{XY} \cong$ ______ $\angle X \cong$ _______

$\overline{YZ} \cong$ ______ $\angle Y \cong$ _______

$\overline{ZX} \cong$ ______ $\angle Z \cong$ _______

$\Delta XYZ \cong$ _______

State the coordinates of X and its corresponding vertex:

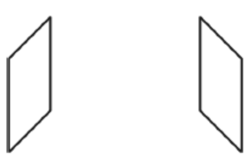
X: ____________ ______________

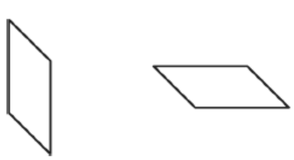
Write the general rule for the translation

____

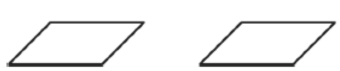
Multiple choice: The following are multiple choice questions. Circle the letter next to the answer.

6. Which pair of shapes shows a translation (slide)?

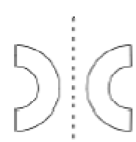
A. 

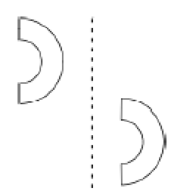
B. 

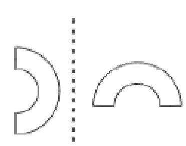
C. 

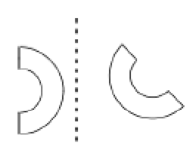
D. 

7. Which shows a slide of  ?

A. 

B. 

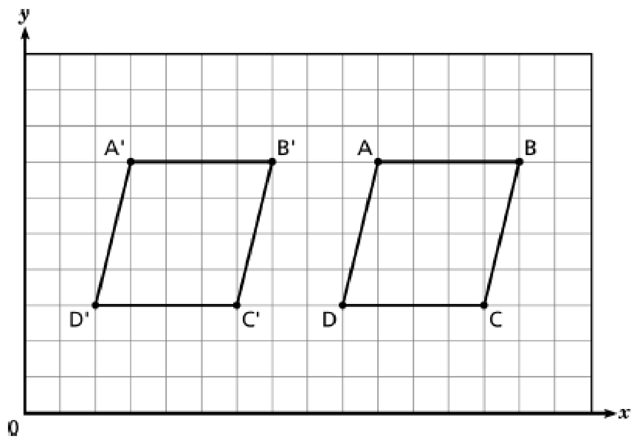
C. 

D. 

Homework is continued on the next page.

Multiple choice: The following are multiple choice questions. Circle the letter next to the answer.

8 Parallelogram *ABCD* was translated to parallelogram *A'B'C'D'*.

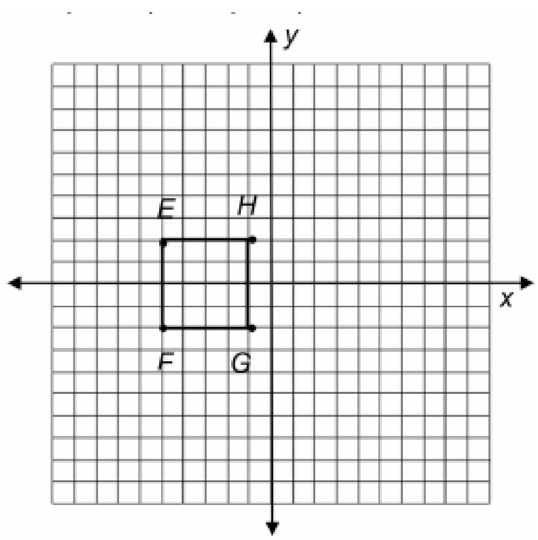


How many units and in which direction were the *x*-coordinates of parallelogram *ABCD* moved?

A. 3 units to the right B. 3 units to the left

C. 7 units to the right D. 7 units to the left

9 Figure *EFGH* in the coordinate plane has vertices at (−5, 2), (−5, −2), (−1, −2), and (−1, 2).



If the figure is translated 5 units to the right and 2 units up, what are the coordinates of the *E'F'G'H'*?

A. (0, 4), (0, 0), (4, 0), (4, 4)

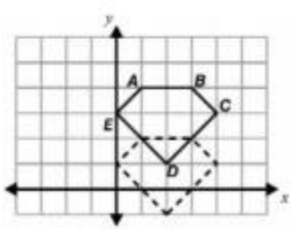
B. (−3, 7), (−3, 3), (1, 3), (1, 7)

C. (−10, 0), (−10, 4), (−6, −4), (−6, 0)

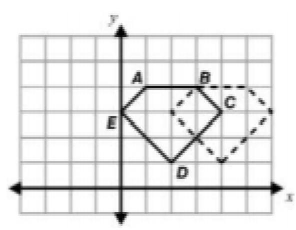
D. (−7, −3), (−7, −7), (−3, −7), (−3, −3)

10. Which shows the translation of pentagon *ABCDE* two units to the left?

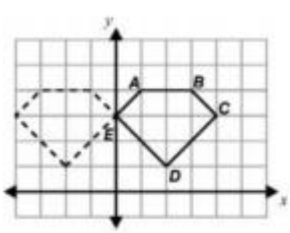
A.



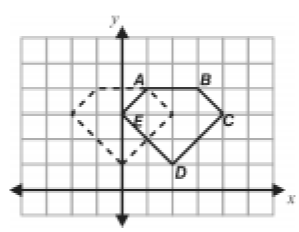
B.



C.



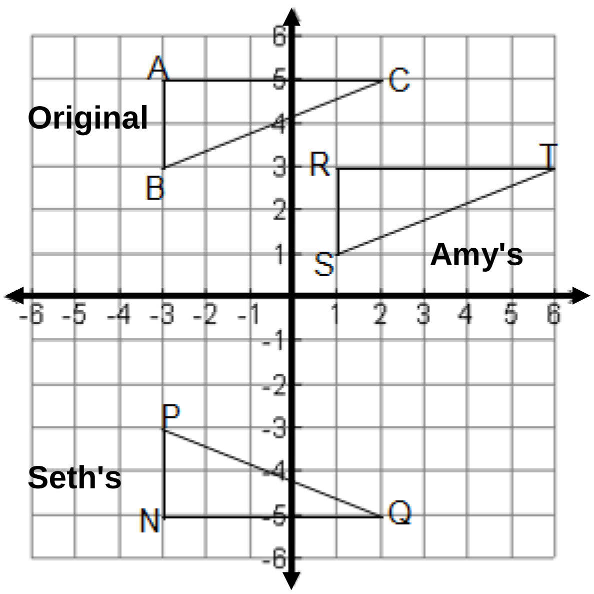
D.



Homework is continued on the next page.

11.

Amy transformed triangle ABC to create triangle RST. State the type of transformation and give the general rule.	Seth transformed triangle ABC to create triangle NPQ. State the type of transformation and give the general rule.
Type: ______________________________	Type: ______________________________
Rule: ______________________________	Rule: ______________________________



Name the corresponding parts for the triangles.

For Amy's transformation...

$\overline{AB} \cong$ ______ $\angle A \cong$ _______

$\overline{BC} \cong$ ______ $\angle B \cong$ _______

$\overline{CA} \cong$ ______ $\angle C \cong$ _______

$\Delta ABC \cong$ _______

For Seth's transformation...

$\overline{AB} \cong$ ______ $\angle A \cong$ _______

$\overline{BC} \cong$ ______ $\angle B \cong$ _______

$\overline{CA} \cong$ ______ $\angle C \cong$ _______

$\Delta ABC \cong$ _______

Multiple choice: The following are multiple choice questions. Circle the letter next to the answer.

12.

Point P has coordinates $(2, 5)$. After a translation, the coordinates of its image P' are $(4, -1)$.

Which of the following best describes the translation?

A. right 1 unit, down 4 units

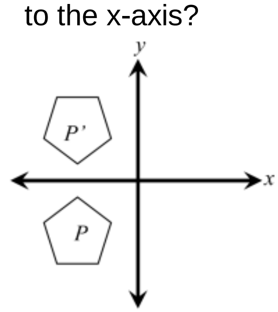
B. right 2 units, down 4 units

C. right 2 units, down 6 units

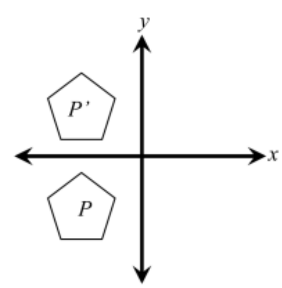
D. right 4 units, down 1 unit

13. Which figure is a reflection of figure P in respect to the x-axis?

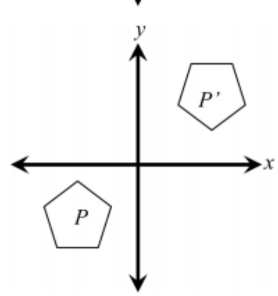
A.



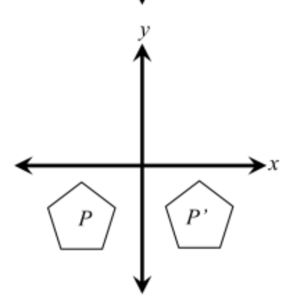
B.



C.

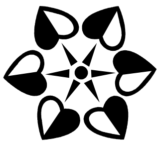
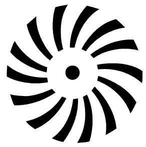
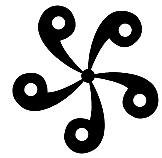


D.



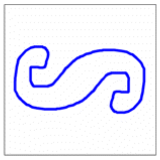
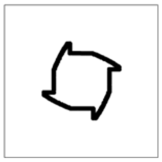
Rotational Symmetry

An image has **<u>Rotational Symmetry</u>** if there is a center point where an object is turned a certain number of degrees and still look the same. A rotation is sometimes called a TURN. These examples have rotational symmetry, but no reflectional symmetry.

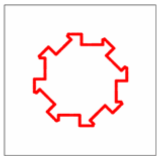
    

How many **matches** there are as you go **once around** is called the **Order**.

<u>Examples of Different Rotational Symmetry Order</u>

Order	**Example Shape**	**Angle of Rotation**
		$360^o \div 2 =$ **180^o**
		$360^o \div 3 =$ **120^o**
		$360^o \div 4 =$ **90^o**

... and there is also Order 5, 6, 7, and ...

		$360^o \div 8 =$ **45^o**

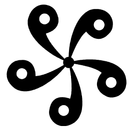
... and then there is Order 9, 10, and so on ...

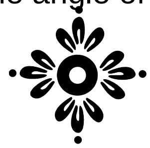
Is there Rotational Symmetry of Order 1 ?

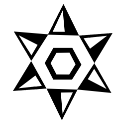
Not really! If a shape only matches itself **once** as you go around (ie it matches itself after one full rotation) there is really no symmetry at all, because the word "Symmetry" comes from *syn-* ***together*** and *metron* ***measure***, and there can't be "together" if there is just one thing.

Practice: For each figure state the order and the angle of rotation.

1. 
Order: _____
Angle : _____________

2. 
Order: _____
Angle : _____________

3. 
Order: _____
Angle : _____________

4. 
Order: _____
Angle : _____________

Notes for Rotational Symmetry on a Coordinate Grid

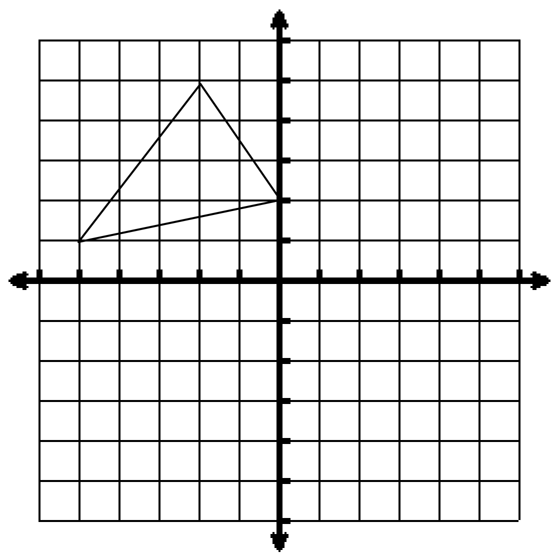
The vertices of a polygon are listed. Graph and label each polygon and its image after a given rotation. Name the coordinates of the image.

1. Rotate figure STU about the origin 90^o clockwise.

S (0, 2) → S' _______

T (-2, 5) → T' _______

U (-5, 1) → U' _______



Write the general rule:

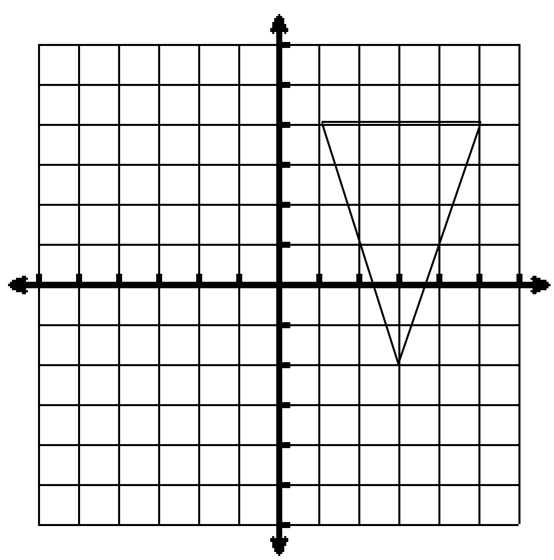
____

2. Rotate figure EFG about the origin 180^o .

E (1, 4) → E' _______

F (3, -2) → F' _______

G (5, 4) → G' _______



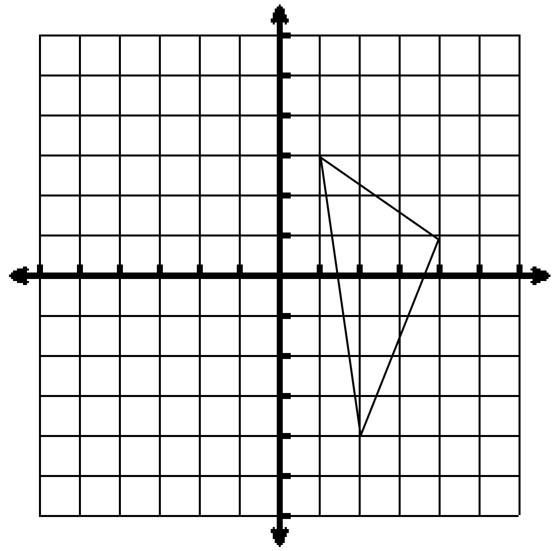
Write the general rule:

____

Homework for Rotational Symmetry

The vertices of a polygon are listed. Graph and label each polygon and its image after a given rotation. Name the coordinates of the image.

1. Rotate figure WXY about the origin 90^o counterclockwise.



W (2,-4) → W' _______

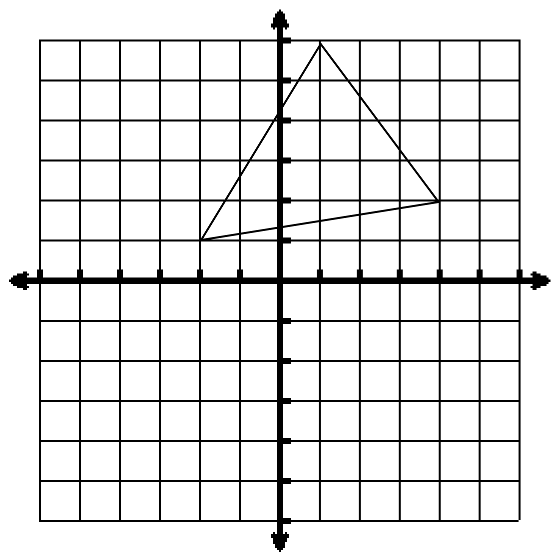
X (1, 3) → X' _______

Y (4, 1) → Y' _______

Write the general rule:

____

2. Rotate figure JKL about the origin 90^o clockwise



J (-2, 1) → J' _______

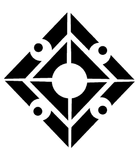
K (1, 6) → K' _______

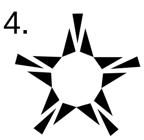
L (4, 2) → L' _______

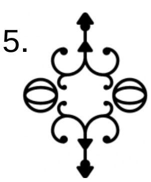
Write the general rule:

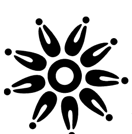
____

For each figure state the order and the angle of rotation.

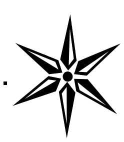
3. 
Order: _____
Angle : ____________

4. 
Order: _____
Angle : ____________

5. 
Order: _____
Angle : ____________

6. 
Order: _____
Angle : ____________

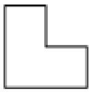
7. 
Order: _____
Angle : ____________

8. 
Order: _____
Angle : ____________

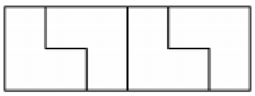
Homework is continued on the next page.

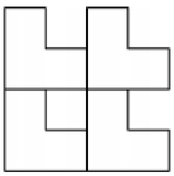
Multiple choice: The following are multiple choice questions. Circle the letter next to the answer.

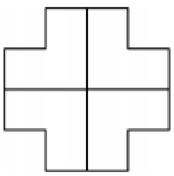
9. Irene is making a tessellation using the shape shown below.

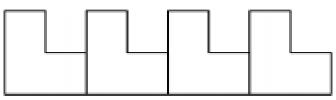


Which of the following tessellations can be made using only a clockwise rotation?

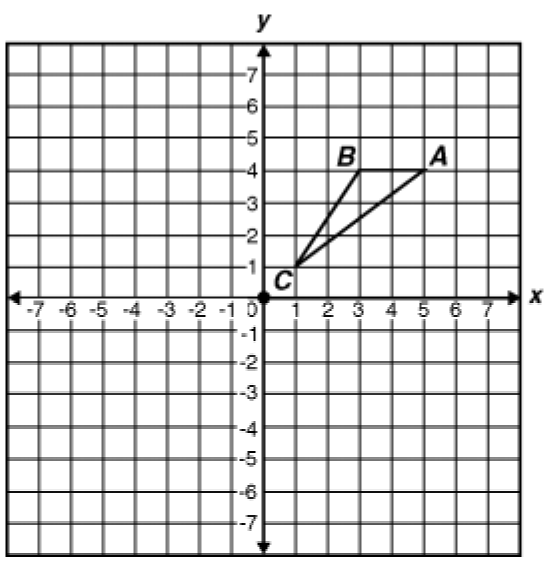
A. 

B. 

C. 

D. 

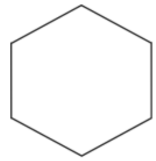
10. If triangle *ABC* is rotated 180 degrees about the origin, what are the coordinates of *A*′?

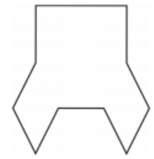


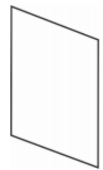
A. (–5, –4) B. (–5, 4)

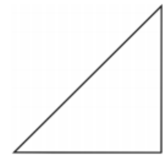
C. (–4, 5) D. (–4, –5)

11. Which figure has a line of symmetry and rotational symmetry?

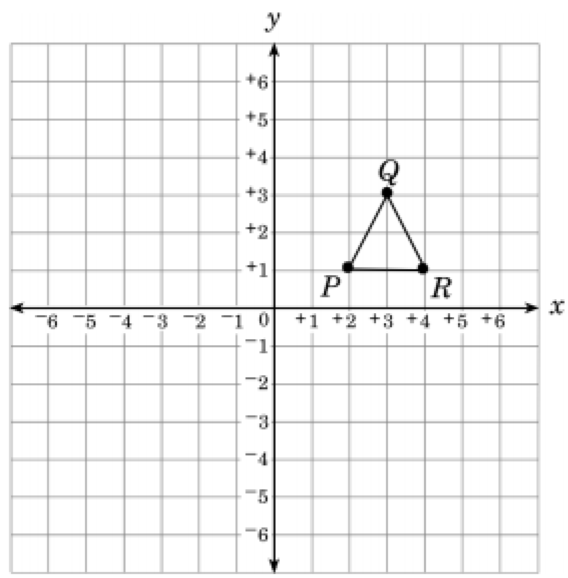
A. 

C. 

B. 

D. 

12. Triangle *PQR* will be rotated 90° counterclockwise about the origin.

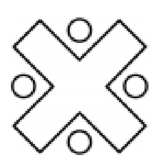


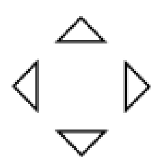
What will be the coordinates of *R*′?

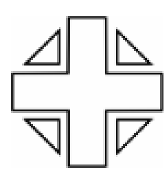
A. (4, 1) B. (0, 4)

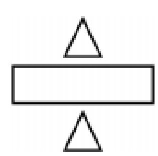
C. (–1, –4) D. (–1, 4)

13. Which figure below has line symmetry but *does not* have rotational symmetry?

A. 

B. 

C. 

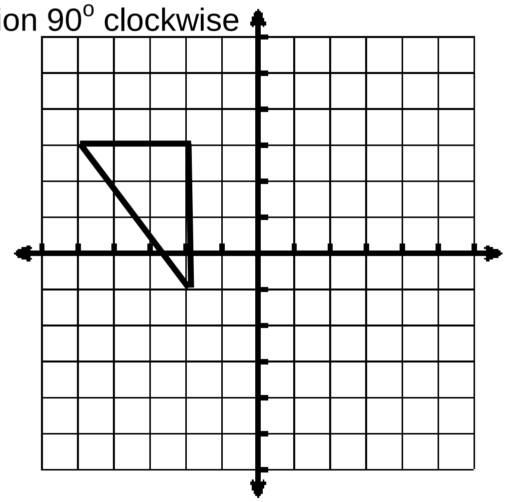
D. 

Homework is continued on the next page.

More Rotational Symmetry

Plot and label the figure on each coordinate grid. Make the transformation that is indicated. State the transformed coordinates and the general rule.

1. Rotation 90^o clockwise



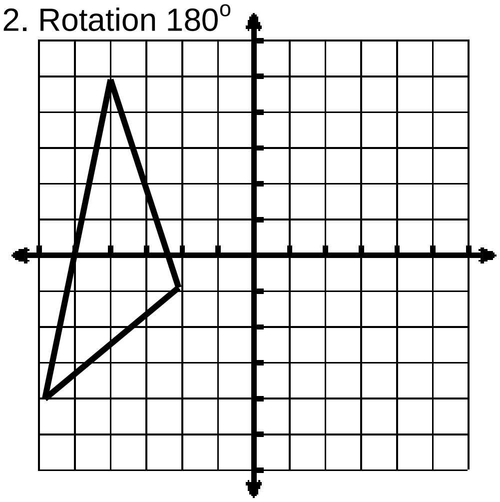
A (-2, -1) A’ _______

B (-2, 3) B’ _______

C (-5, 3) C’ _______

General Rule: _________________

2. Rotation 180^o



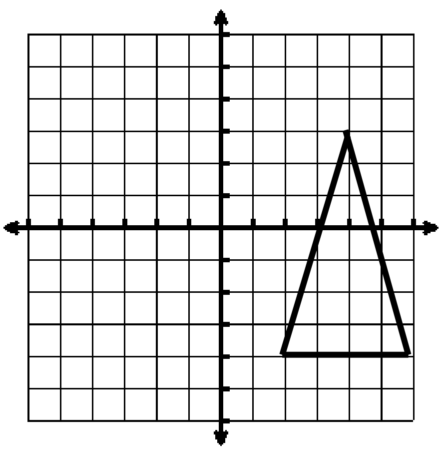
D (-4, 5) D’ _______

E (-6, -4) E’ _______

F (-2, -1) F’ _______

General Rule: _________________

3. Rotation 90^o counterclockwise



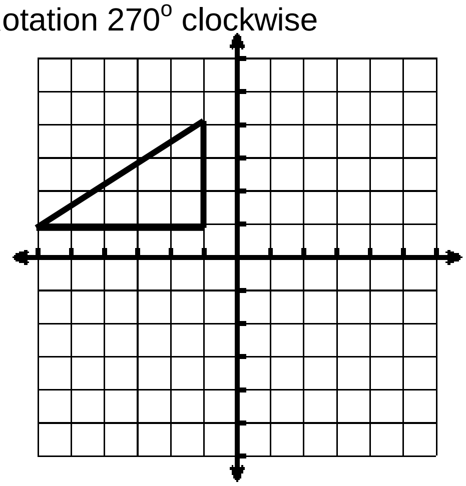
G (2, -4) G’ _______

H (4, 3) H’ _______

I (6, -4) I’ _______

General Rule: _________________

4. Rotation 270^o clockwise



J (-1, 4) J’ _______

K (-6, 1) K’ _______

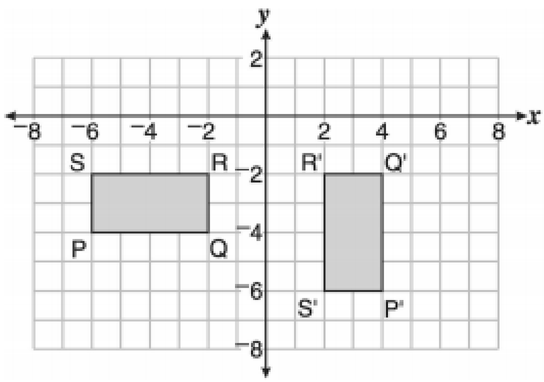
L (-1, 1) L’ _______

General Rule: _________________

Homework is continued on the next page.

Multiple c hoice: The following are multiple choice questions. Circle the letter next to the answer.

5. The figure below depicts a coordinate plane, rectangle PQRS, and the image of rectangle PQRS after a transformation. Point P' is the image of point P, Q' is the image of Q, R' is the image of R, and S' is the image of S.



Which transformation produced the image P'Q'R'S'?

A. a 180-degree counterclockwise rotation about the point (0, 0)

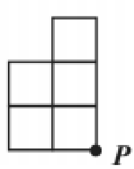
B. a translation of four units to the right

C. a 90-degree counterclockwise rotation about the point (0, 0)

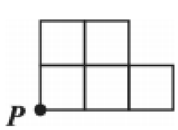
D. a reflection over the *y*-axis

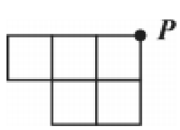
6. Betty drew the figure shown below.

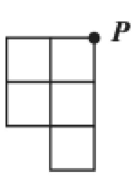
Betty's Figure

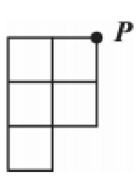


Which of the following shows Betty's figure after it has been rotated 90° *clockwise* about point *P*?

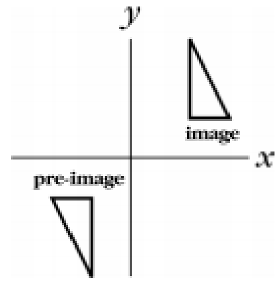
A.


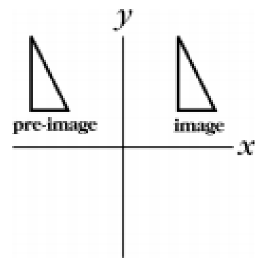
B.


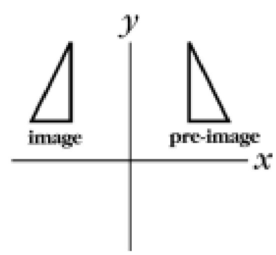
C.


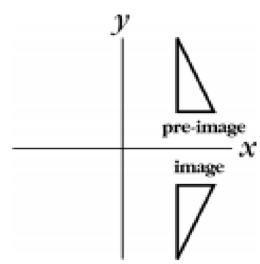
D.


7. Which diagram below best shows a *rotation* of the pre-image to the image?

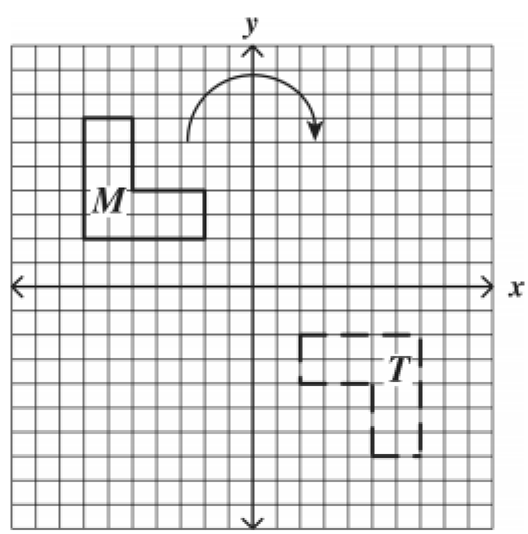
A.


C.


B.


D.


8. In the graph below, figure *M* was rotated clockwise about the origin to generate figure *T*.



What was the angle of rotation of figure *M* about the origin?

A. 90°

B. 180°

C. 270°

D. 360°

Homework is continued on the next page.

9. A polygon has been rotated about the origin. Which statement must be true?

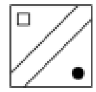
A. The lengths of the sides are doubled.

B. The area of the polygon did not change.

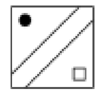
C. The coordinates of the vertices did not change.

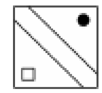
D. The area of the polygon is 4 Times its original area.

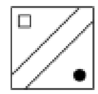
10. The following figure is to be rotated 90° clockwise.

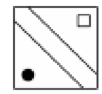


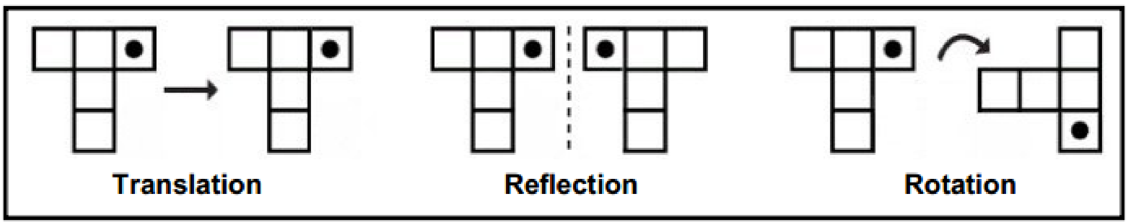
What will the figure look like after the rotation?

A. 

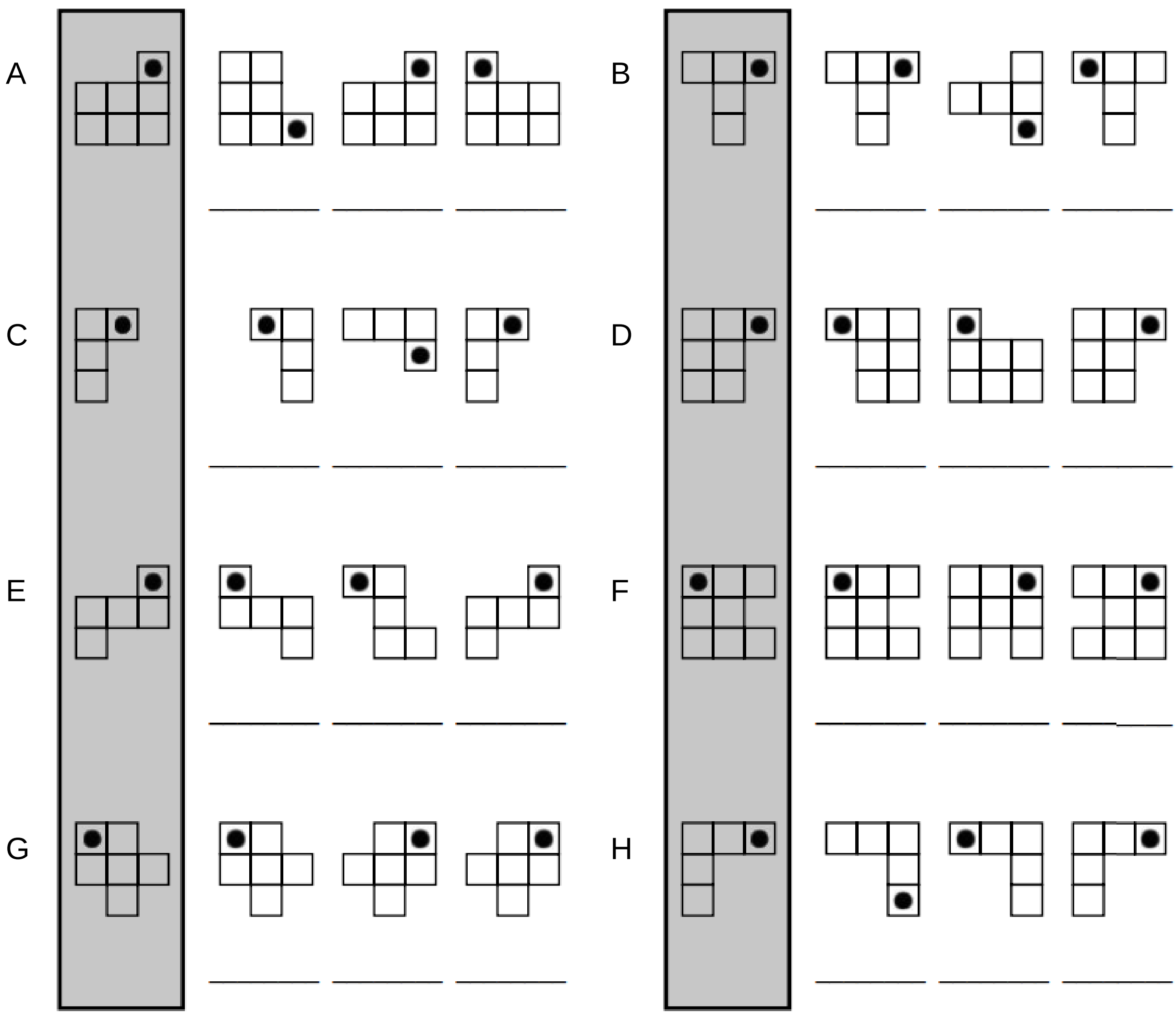
B. 

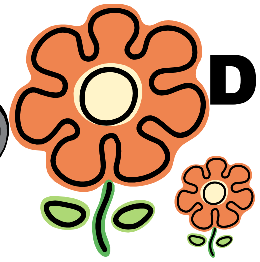
C. 

D. 



11. Identify each shape as translation, rotation, and reflection.



Dilations

Dilations

A transformation in which a polygon is enlarged or reduced by a given factor around a given center point.

Dilation is where the polygon grows or shrinks but keeps the same overall shape. It's a little like zooming in or out on a camera.

The transformed figure is called the dilated image of the original

Scale factor

The amount by which the image grows or shrinks is called the "Scale Factor".

- If the scale factor is say 2, the image is enlarged to twice the size of the original.
- If it is 0.5, the image is reduced to half the size.
- When the scale factor is 1, the image is the exact same size as the original.

Remember: In dilation, **multiply** the dimensions of the original by the scale factor to get the dimensions of the image.

Original and image are similar

In dilation, the image and the original are <u>similar</u>, in that they are the same shape but not necessarily the same size. They are **not** <u>congruent</u> because that requires them to be the same shape **and** the same size, which they are not (unless the scale factor happens to be 1.0).

<u>NOTES for Dilations</u>

1. Dilate figure WXY by a **scale factor of 2**.

Plot and label the original and the dilated figure.

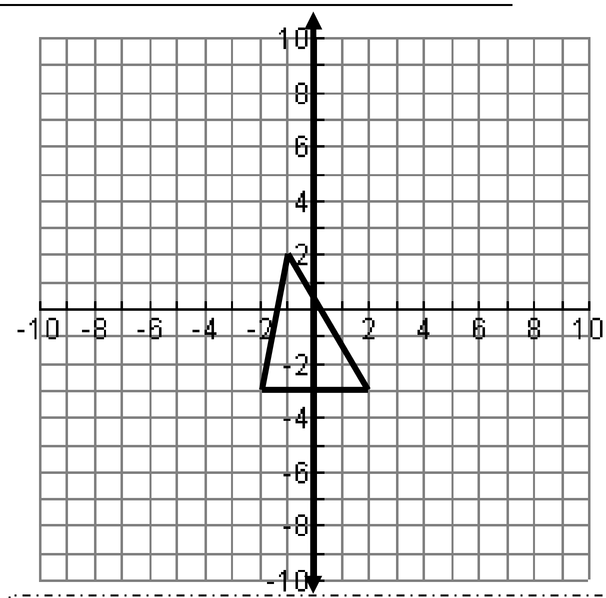
W (-1, 2) → W' _______

X (-2, -3) → X' _______

Y (2, -3) → Y' _______

Find the area of the original figure: ________

Find the area of the dilated figure: ________

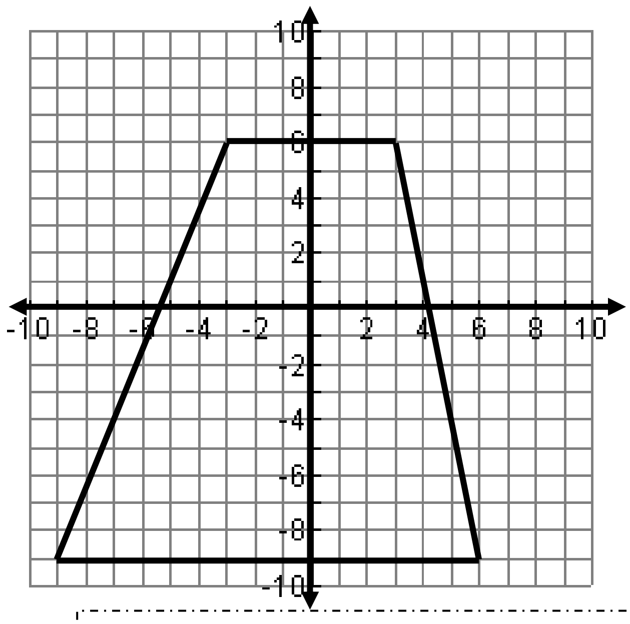


Write a general rule for the dilation:

____

NOTES for Dilations

2. Dilate figure ABCD by a **scale factor of $\frac{1}{3}$.**
Plot and label the original and the dilated figure.



A (-3, 6) → A’ _______

B (-9, -9) → B’ _______

C (6, -9) → C’ _______

D (3, 6) → D' _______

Find the area of the original figure: ________

Write a general rule for the dilation:

____

Find the area of the dilated figure: ________

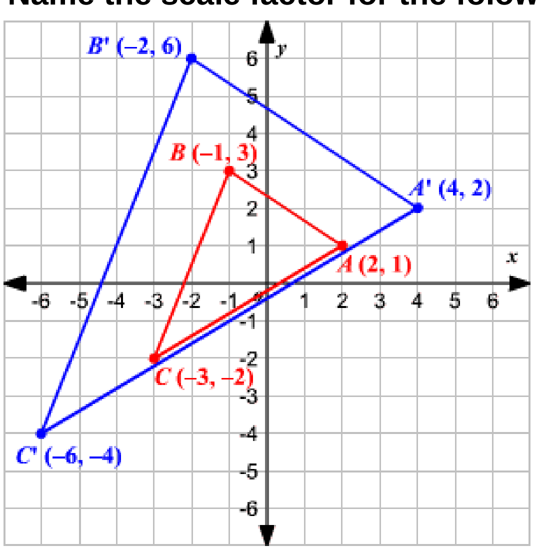
State the scale factor of the following dilations:

3. (2, 4) → (10, 20) ____ 4. (-15, 27) → (-5, 9) _____ 5. (3, 7) → (12, 28) _____

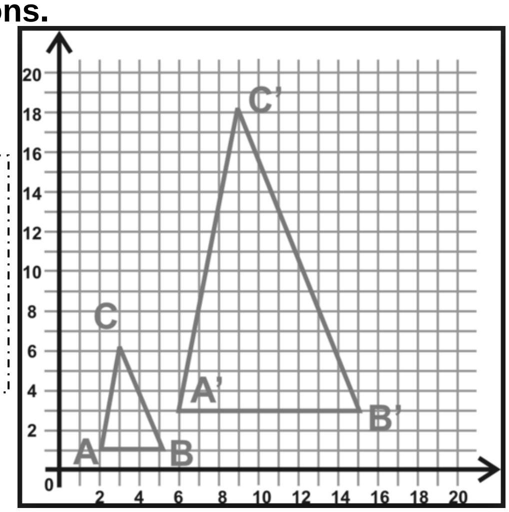
Write the general rule for the transformation.

6. (14, 6) → (7, 3) _________________ 7. (-1, 3) → (-5, 15) __________________

Name the scale factor for the folowing dilations.



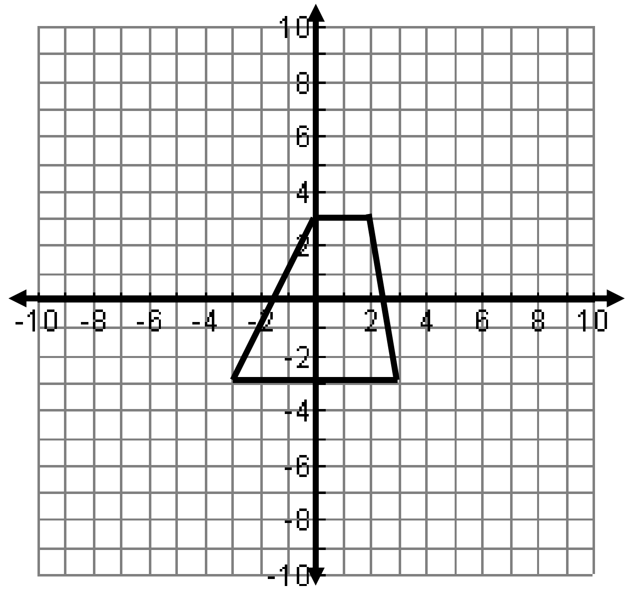
#8. Scale Factor: ______



#9 Scale Factor: ______

Homework for Dilations

1. Dilate figure WXYZ by a **scale factor of 3**.
Plot and label the original and the dilated figure.



W (0, 3) → W' ________

X (2, 3) → X' ________

Y (3, -3) → Y' ________

Z (-3, -3) → Z' ________

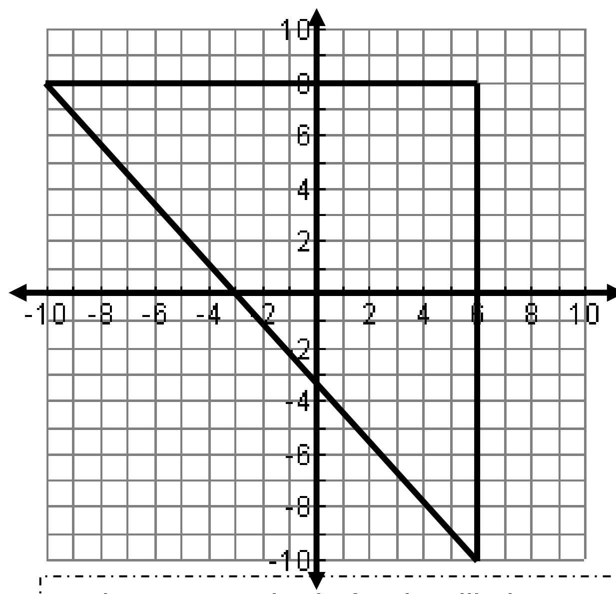
Find the area of the original figure: _________

Find the area of the dilated figure: _________

Write a general rule for the dilation:

____

2. Dilate figure ABC by a **scale factor of $\frac{1}{2}$**.
Plot and label the original and the dilated figure.



A (-10, 8) → A' ________

B (6, 8) → B' ________

C (6, -10) → C' ________

Find the area of the original figure: _________

Find the area of the dilated figure: _________

Write a general rule for the dilation:

____

State the scale factor of the following dilations:

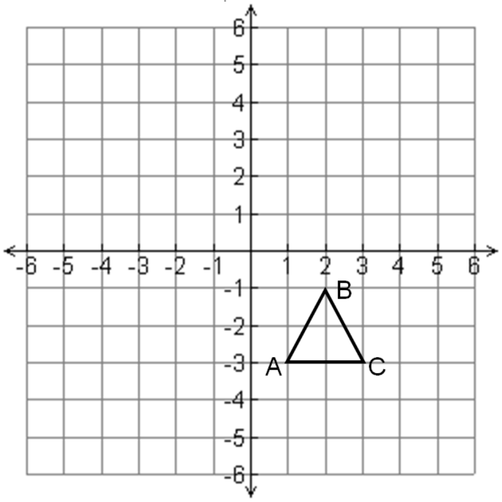
3. (3, 4) → (12, 16) _____ 4. (-15, 50) → (-3, 10) ______ 5. (1, 9) → (6, 54) ______

Write the general rule for the dilation.

6. (21, 6) → (7, 2) __________________ 7. (2, 15) → (4, 30) ___________________

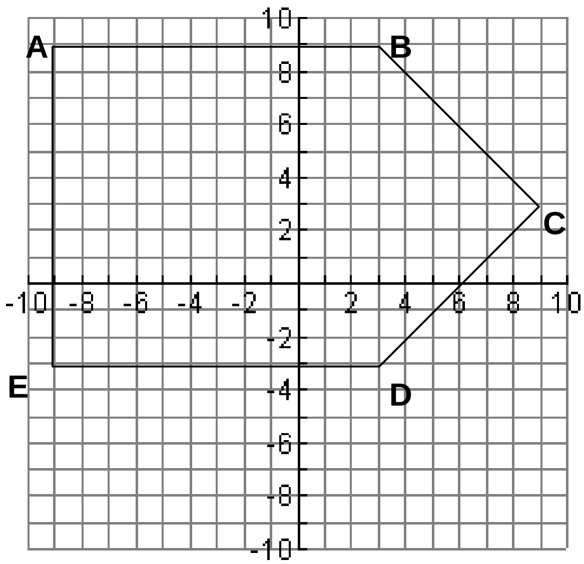
Homework is continued on the next page.

8. Draw the dilation image of triangle ***ABC*** with the center of dilation at the origin and a scale factor of 2. (Hint: write down the coordinates first.)



Is the dilation an enlargement or reduction?

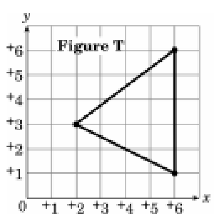
9. Draw the dilation image of pentagon ***ABCDE*** with the center of dilation at the origin and a scale factor of $\frac{1}{3}$. (Hint: write down the coordinates first.)

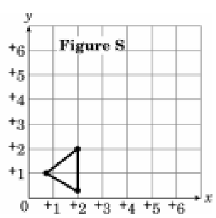


Is the dilation an enlargement or reduction?

Multiple choice: The following are mu ltiple choice questions. Circle the letter next to the answer.

10. Figure *S* is the result of a dilation of Figure *T*.





What is the scale factor of the dilation?

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. 2 D. 3

11. A triangle has the following vertices: $(-1, 1)$, $(6, -2)$, and $(3, 5)$. If the triangle undergoes a dilation with a scale factor of 3, what will be the vertices of the image?

A. $(-3, 3)$, $(18, -6)$, $(9, 15)$

B. $(3, 3)$, $(18, 6)$,$(9, 15)$

C. $(-3, 3)$, $(18, 6)$, $(9, 15)$

D. $(3, 3)$, $(18, -6)$, $(9, 15)$

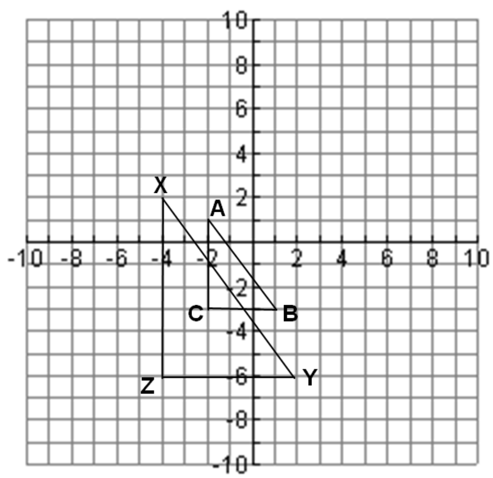
Dilations and Similar Figures

Under a transformation of a dilation, a figure will be similar to the pre-image. This means…

- **the angle measures will remain the same (be congruent)**
- **parallel lines remain parallel**
- **BUT lengths of segments are NOT congruent, but be in equal ratio**

Note:
$\cong$ ***means congruent to***
$\sim$ ***means similar to***

Triangle ABC was dilated by a factor of 2 to create triangle XYZ



$\Delta ABC \sim \Delta XYZ$

$\overline{CA} = 4 units$ $\qquad$ $\overline{XZ} =$ _____ $units$

$\overline{BC} = 3 units$ $\qquad$ $\overline{YZ} =$ _____ $units$

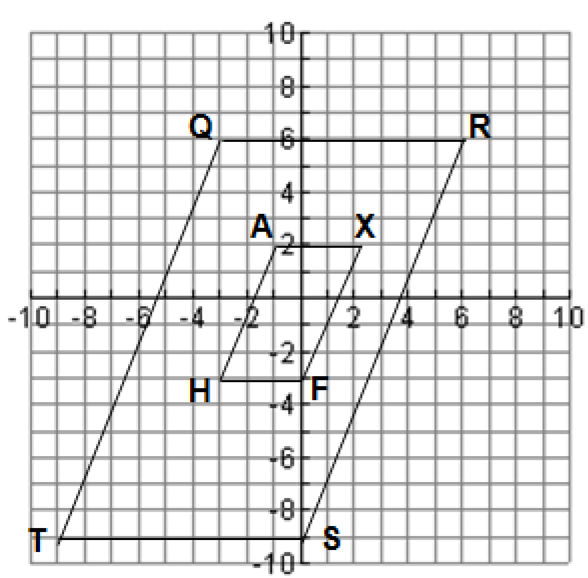
$\overline{AB} = 5\ units$ $\qquad$ $\overline{XY} =$ _____ $units$

Name the congruent angles.

$\angle A \cong$ ______ $\quad$ $\angle B \cong$ ______ $\quad$ $\angle C \cong$ _______

Notice the ratio of all the segment measures remains the same.

$Parallelogram\ QRST$ was dilated by a scale factor of $\frac{1}{3}$. Fill in the missing values.



$Parallelogram\ QRST \sim Parallelogram$__________

$\overline{QR} =$ _____ $units$ $\qquad$ $\overline{AX} =$ _____ $units$

$\overline{TS} =$ _____ $units$ $\qquad$ $\overline{HF} =$ _____ $units$

$\overline{QT} \approx 16\ units$ $\qquad$ $\overline{AH} \approx$ _____ $units$

$\overline{SR} \approx 16\ units$ $\qquad$ $\overline{FX} \approx$ _____ $units$

Name the congruent angles in the smaller parallelogram.

$\angle Q \cong$ _____ $\quad$ $\angle R \cong$ _____ $\quad$ $\angle S \cong$ ______ $\quad$ $T \cong$ ______

If $\overline{QR} \parallel \overline{TS}$, then $\overline{AX} \parallel \overline{HF}$. Therefore if $\overline{QT} \parallel \overline{RS}, then$ name two other parallel segments.

____

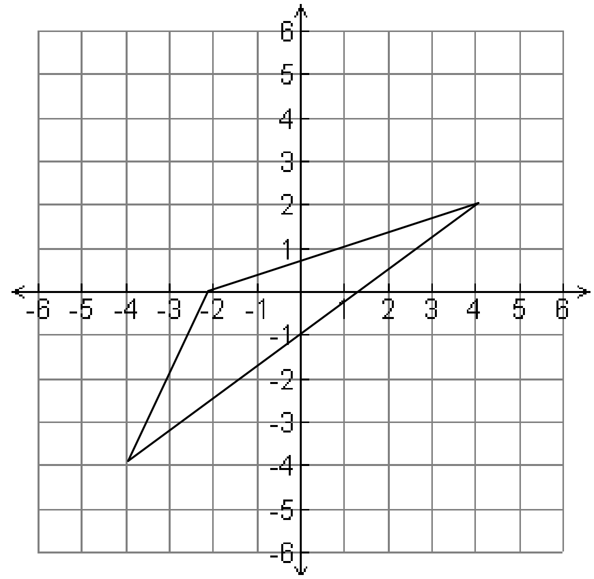
Dilate figure ABC by a **scale factor of $\frac{3}{2}$.**
Plot and label the original and the dilated figure.

A (-2, 0) → A' ________

B (4, 2) → B' ________

C (-4, -4) → C' ________

State the general rule: ______________________



Homework

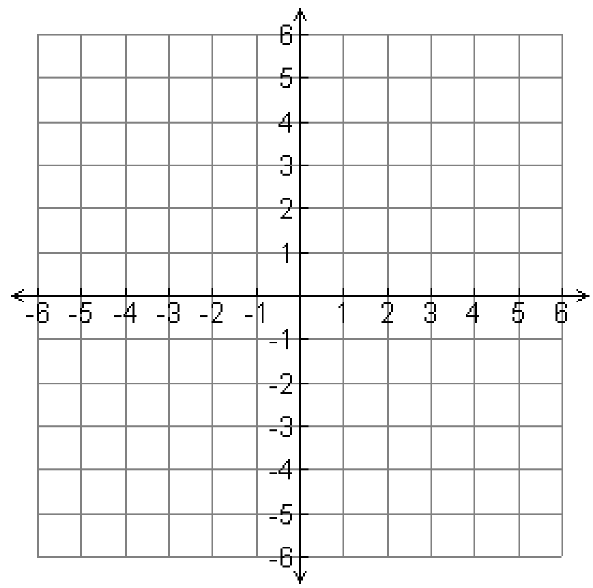
1) Dilate figure ABC by a **scale factor of 2.**
Plot and label the original and the dilated figure.

A (-2, 1) → A' ________

B (-2, 3) → B' ________

C (3, 2) → C' ________

State the general rule: ______________________



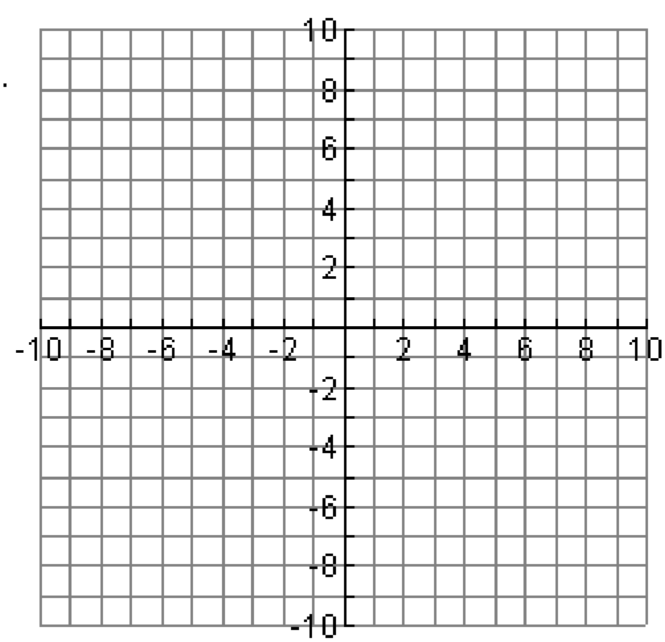
2) Dilate figure ABC by a **scale factor of $\frac{1}{2}$.**
Plot and label the original and the dilated figure.

A (-10, 8) → A' ________

B (-8, 6) → B' ________

C (-6, -10) → C' ________

State the general rule: ______________________



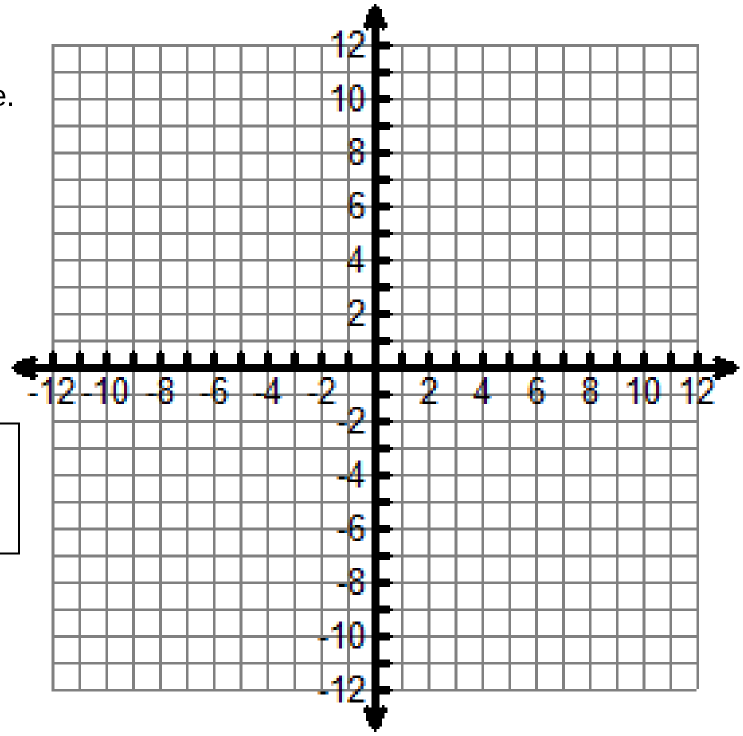
Homework is continued on the next page.

3) Dilate figure ABC by a **scale factor of $\frac{2}{3}$.**
Plot and label the original and the dilated figure.

A (-12, 9) → A' ________

B (9, 6) → B' ________

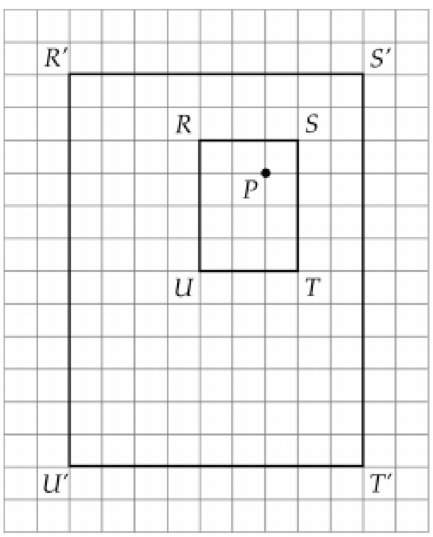
C (6, -12) → C' ________



State the general rule: ____________________

Multiple choice: The following are multiple choice questions. Circle the letter next to the answer.

1. A dilation with center *P* maps the rectangle *RSTU* to the rectangle *R'S'T'U'* as shown below.



What is the scale factor of this dilation?

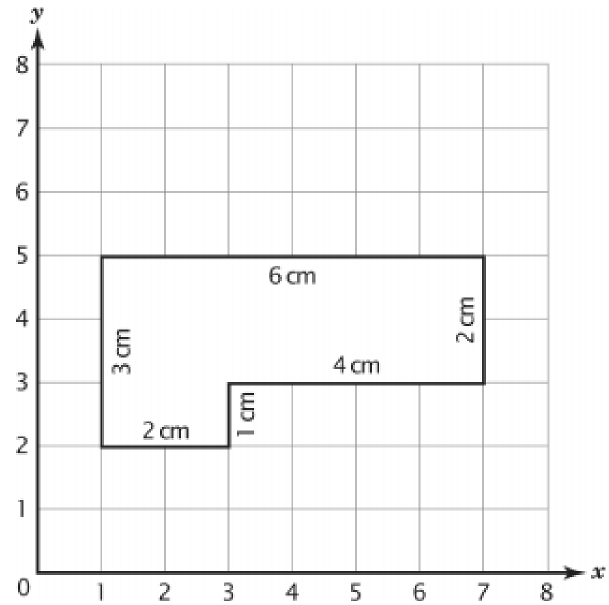
A. 2 B. 3 C. 4 D. 9

2. Which of these transformations can change the area of a polygon?

A. translation B. rotation C. reflection D. dilation

Homework is continued on the next page.

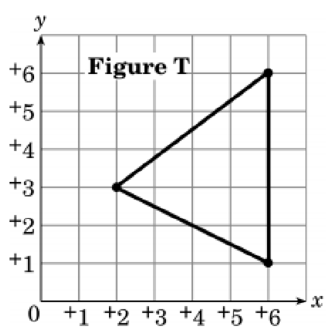
3. Look at the figure on the grid below.

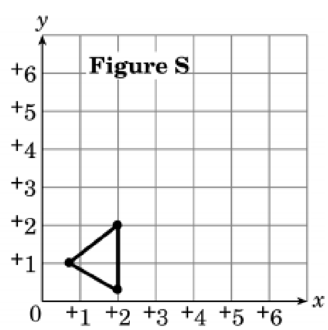


What is the perimeter of the figure after it is dilated (magnified) by a scale factor of 3?

A. 6 centimeters B. 21 centimeters C. 36 centimeters D. 54 centimeters

4. Figure *S* is the result of a dilation of Figure *T*.

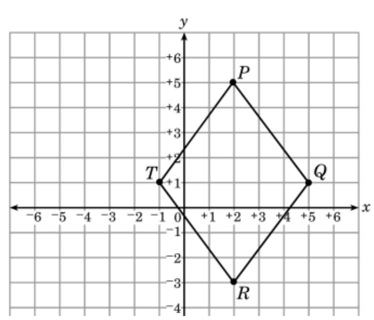




What is the scale factor of the dilation?

A. $\frac{1}{3}$ B. $\frac{1}{2}$ C. 2 D. 3

5. Rhombus *PQRT* is shown

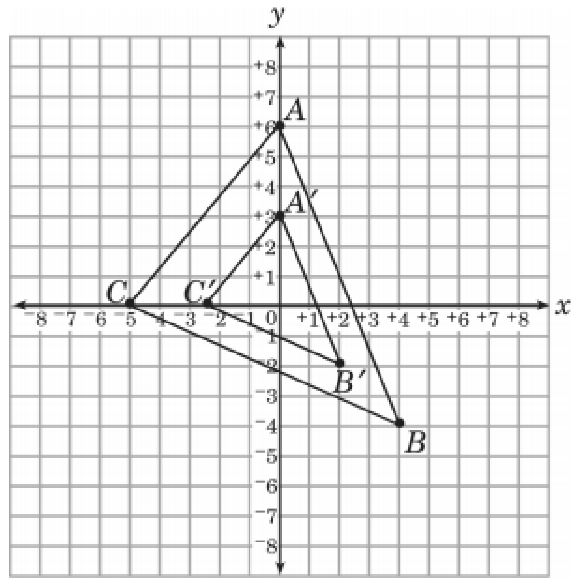


P'Q'R'T' is the image produced by dilating *PQRT* by a scale factor of 4. What is the length of the diagonal *P'R'*?

A. 2 units B. 8 units C. 12 units D. 32 units

Homework is continued on the next page.

6. In the figure shown below, $\triangle A'B'C'$ is the image produced by applying a dilation to $\triangle ABC$.



What is the scale factor for this dilation?

A. $\frac{1}{3}$ B. $\frac{2}{5}$ C. $\frac{1}{2}$ D. $\frac{5}{2}$

7. A point has the coordinates (4, 8). The point will be dilated by a scale factor of 2. What will be the coordinates of the image point?

A. (6, 8) B. (8, 16) C. (24, 28)

8. Triangle ABC has vertices at $A(2, 2)$, $B(2, 7)$, and $C(6, 3)$. This triangle is dilated by a scale factor of 3. What is the location of point C'?

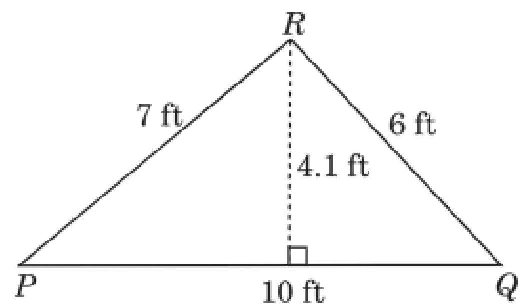
A. (2, 1) B. (6, 6) C. (6, 21) D. (18, 9)

9. The vertices of a rectangle are (0, 0), (0, 4), (2, 4), (2, 0). Which of the following points is a vertex for the image produced by a dilation with a scale factor of $\frac{1}{2}$?

A. (0, 3) B. (0, 2) C. (0, 1) D. (2, 1)

Homework is continued on the next page.

10. What is the perimeter of a triangle whose dimensions are three times the size of $\triangle PQR$?



A. 23 ft
B. 27.1 ft
C. 69 ft
D. 81.3 ft

11. Sonya plans to use a copy machine to dilate the sign

If she uses a scale factor of $\frac{1}{4}$, which statement describes how the sign's diameter will change after the dilation?

A. The diameter will be 4 times longer.
B. The diameter will be $\frac{1}{4}$ as long.
C. The diameter will be 2 times longer.
D. The diameter will be $\frac{1}{2}$ as long.

12. $\triangle GHJ$ with vertices $G(-2, 4)$, $H(3, 6)$,and $J(3, -2)$ is dilated by a factor of $\frac{1}{3}$. What are the coordinates of the vertex of the image $\triangle G'H'J'$ that lies in the second quadrant?

A. $\left(\frac{-7}{3}, \frac{13}{3}\right)$
B. $\left(\frac{-2}{3}, \frac{4}{3}\right)$
C. $\left(1, \frac{-2}{3}\right)$
D. $(1, 2)$

13. $\triangle GHI$ will be dilated by a scale factor of 3, resulting in $\triangle G'H'I$. What rule describes this transformation?

A. $(x', y') = (\frac{1}{3}x, \frac{1}{3}y)$
B. $(x', y') = (3x, 3y)$
C. $(x', y') = (x + 3, y + 3)$
D. $(x', y') = (x - 3, y - 3)$

14. $\triangle XYZ$ is dilated by a factor of $\frac{1}{2}$. What is the ratio of the area of $\triangle XYZ$ to the area of its image, $\triangle X'Y'Z'$?

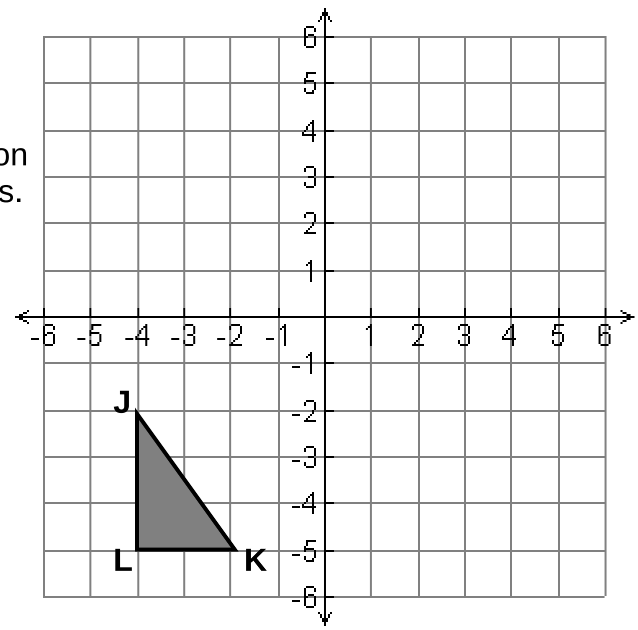
A. 4 : 1
B. 2 : 1
C. 1 : 2
D. 1 : 4

Combined Transformations

A combined transformation is just a series of two or more transformations performed on the same figure.

EXAMPLES of Double Transformations

1. Using triangle JKL, find each point of reflection over the y-axis and then a translation up 5 units.



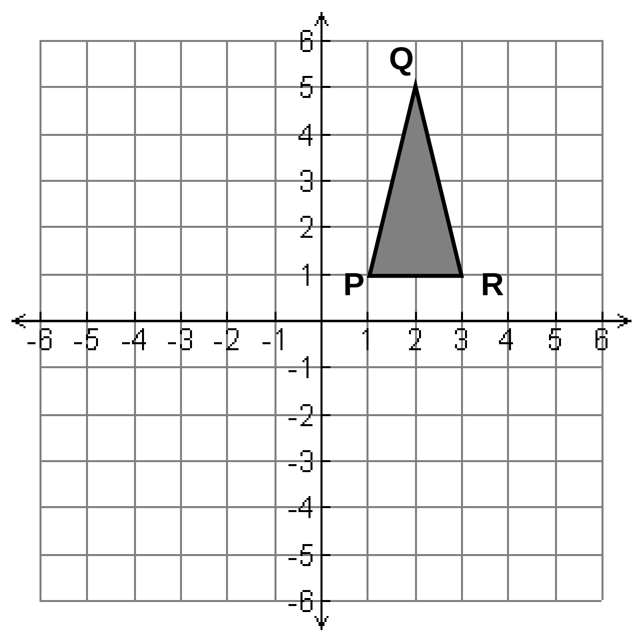
J ______ J' ______ J'' ______

K ______ K' ______ K'' ______

L ______ L' ______ L'' ______

Draw triangle J'K'L' and J''K''L''

2. Using figure PQR, find each point for a rotation 180^{o} about the origin and a translation right 5 units and up 1 units.



P ______ P' ______ P'' ______

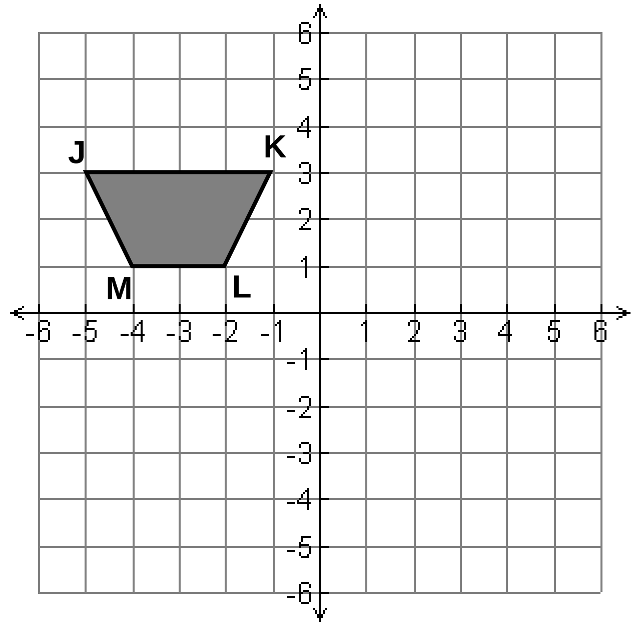
Q ______ Q' ______ Q'' ______

R ______ R' ______ R'' ______

Draw triangle P'Q'R' and P''Q''R''

Homework on Combined Transformations

1. Using figure JKLM, find each point for a reflection over the x-axis and a translation down 3 units.



J ______ J' ______ J'' ______

K ______ K' ______ K'' ______

L ______ L' ______ L'' ______

M ______ M' ______ M'' ______

Draw figure J'K'L'M' and J''K''L''M''

Find the area of figure JKLM. Show all work.

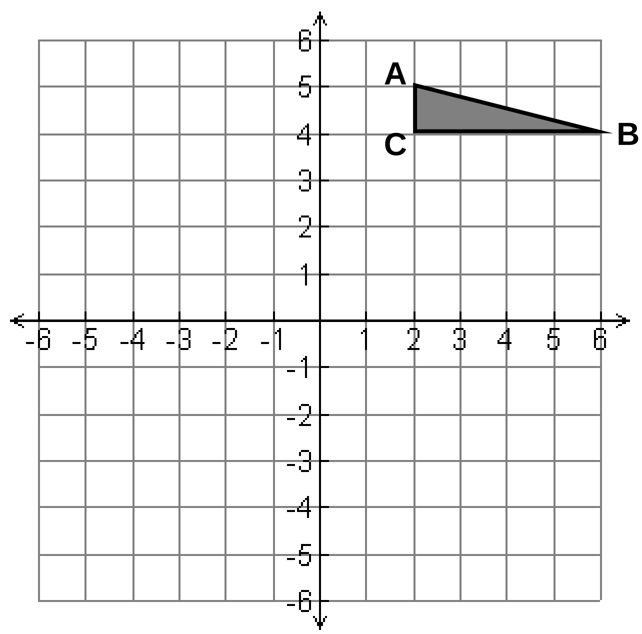
Area: ______

2. Using figure ABC, find each point for a translation left 2 and down 3 and then a rotation of 90^{o}counterclock wise.

A ______ A' ______ A'' ______

B ______ B' ______ B'' ______

C ______ C' ______ C'' ______



Draw triangle A'B'C' and A''B''C''

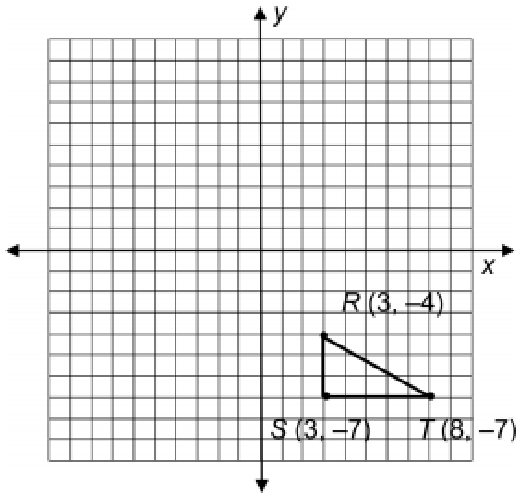
Find the area of figure ABC. Show all work.

Area: ______

Homework is continued on the next page.

Multiple choice: The following are multiple choice questions. Circle the letter next to the answer

3. Triangle *RST* is shown in the coordinate plane.



What are the coordinates of *R′S′T′* if the figure is reflected over the *x*-axis and translated down two units?

A. (1, –6), (1, –9), (6, –9)

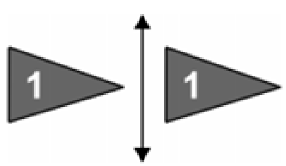
B. (3, 4), (3, 7), (8, 7)

C. (1, 2), (1, 5), (6, 5)

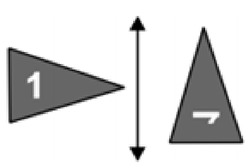
D. (3, 2), (3, 5), (8, 5)

4. Which figure shows the flag on the left after it has been flipped across the line and then rotated 90^{o} clockwise?

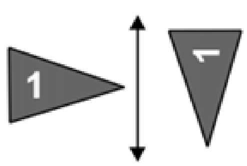
A.



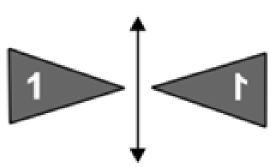
B.



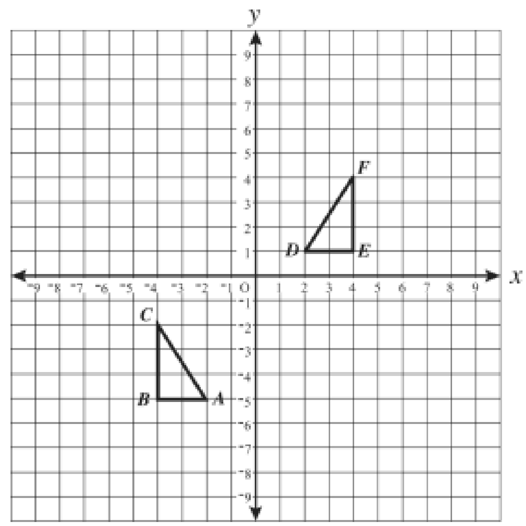
C.



D.



5. △*ABC* and △*DEF* are shown on the grid below.



Which of the following transformations will map △*ABC* onto △*DEF*?

A. Reflect △*ABC* over the *y*-axis and shift up 6 spaces.

B. Reflect △*ABC* over the *x*-axis and shift up 6 spaces.

C. Reflect △*ABC* over the *y*-axis and shift down 6 spaces.

D. Reflect △*ABC* over the *y*-axis, reflect over the *x*-axis, and shift down 4 spaces.

6. Three transformations will be performed on triangle *ABC*. Which set of transformations will always produce a congruent triangle?

A. dilation, rotation, translation

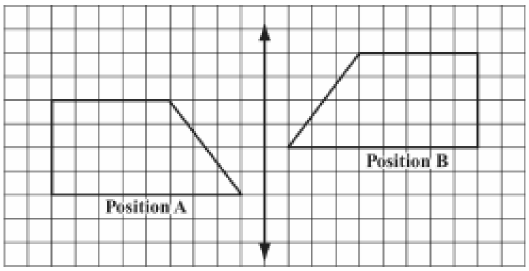
B. reflection, dilation, translation

C. rotation, reflection, dilation

D. rotation, translation, reflection

Homework is continued on the next page.

7. A shape was moved from Position *A* to Position *B*, as shown below.



Which of the following best describes how the shape was moved from Position *A* to Position *B*?

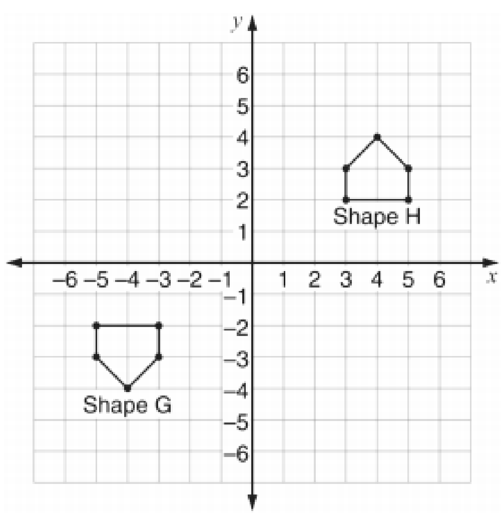
A. flipped over the line, then slid up

B. flipped over the line, then slid down

C. flipped over the line, then turned 90° clockwise

D. flipped over the line, then turned 90° counterclockwise

8. Look at Shape G and Shape H on this grid.



Which transformations will show that Shape G is congruent to Shape H?

A. Translate Shape G right 8 units and then reflect it across the *y*-axis.

B. Translate Shape G right 6 units and then reflect it across the *x*-axis.

C. Translate Shape G right 8 units and then reflect it across the *x*-axis.

D. Translate Shape G up 6 units and then reflect it across the *y*-axis.

9.

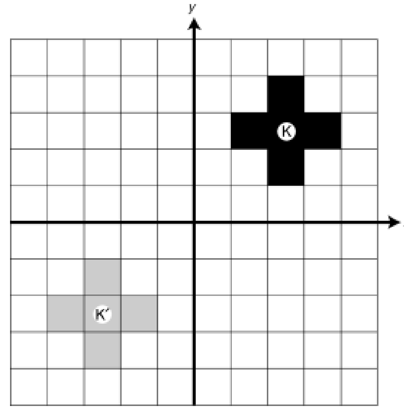


Figure K′ is the result of a sequence of transformations of Figure K. Which of the following does *not* describe a correct possible sequence of transformations?

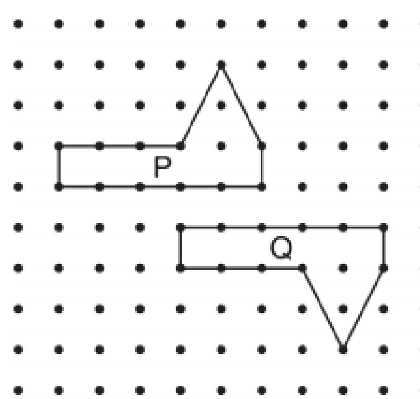
A. a translation of Figure K down 5 units, then a translation to the left 5 units

B. a reflection of Figure K across the *x*-axis, then a translation to the left 5 units

C. a reflection of Figure K across the *y*-axis, then a translation down 4 units

D. a reflection of Figure K across the *x*-axis, then a reflection across the *y*-axis

10. Look at Figure P and Figure Q.



Which motion or motions will result in Figure P exactly covering Figure Q?

A. slides only

B. turns only

C. flips and turns only

D. flips and slides only

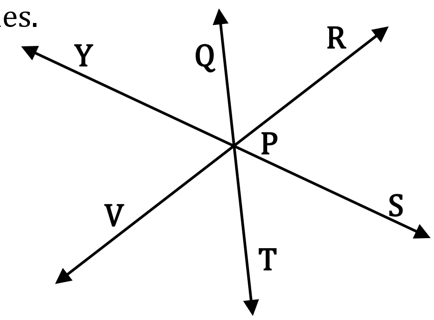
Vertical, Complementary, and Supplementary Angles

When two lines intersect, **two pairs** of **VERTICAL ANGLES** are formed. Vertical angles are <u>**not**</u> adjacent. Vertical angles are located across from each other, they share a common vertex, and the sides of the angles are composed of opposite rays.

Pairs of vertical angles always have the same measure. Vertical angles are congruent (symbol ≅) Congruent means they have the ______________ ____________________________.

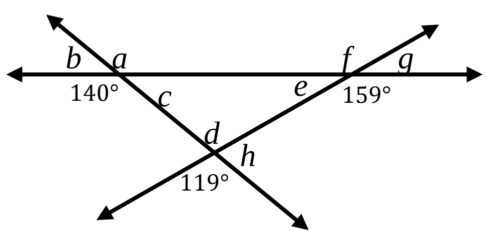
In the diagram, name the second angle in each pair of vertical angles.

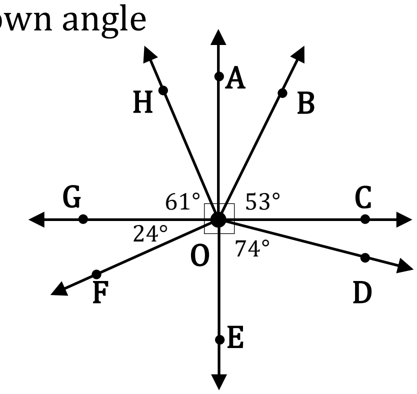
1) ∡YPV _____________	4) ∡VPT _____________
2) ∡QPR _____________	5) ∡RPT _____________
3) ∡SPT _____________	6) ∡VPS _____________



Two angles are **complementary** if the sum of their angles measure 90°.
Two angles are **supplementary** if the sum of their angles measure 180°.
Complementary and supplementary angle pairs *may be* adjacent, but *do not need to be*.

<u>**PRACTICE:**</u> Calculate the measure of each unknown angle





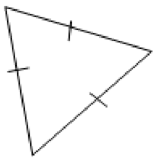
The sum of all the central angles =

SET A	**The sum of angles e + d + c =**
1) m∡ *a* = _____________	5) m∡ *e* = ______________
2) m∡ *b* = ______________	6) m∡ *f* = ______________
3)m∡ *c* = ______________	7) m∡ *g* = ______________
4) m∡ *d* = _____________	8) m∡ *h* = ______________

SET B
9) m∡ AOB = ______________
10) m∡ COD = ______________
11)m∡ EOF = _______________
12) m∡ AOH = ______________

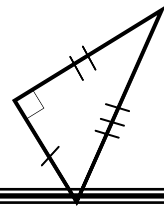
Review: Lines and Angles

Notes: **Identify each type of triangle by its angles and by its sides**



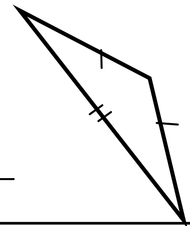
By sides: ______________

By angles: ______________



By sides: ____________

By angles: ____________



By sides: ____________

By angles: _____________

Part 1: Find the measure of the angles below.

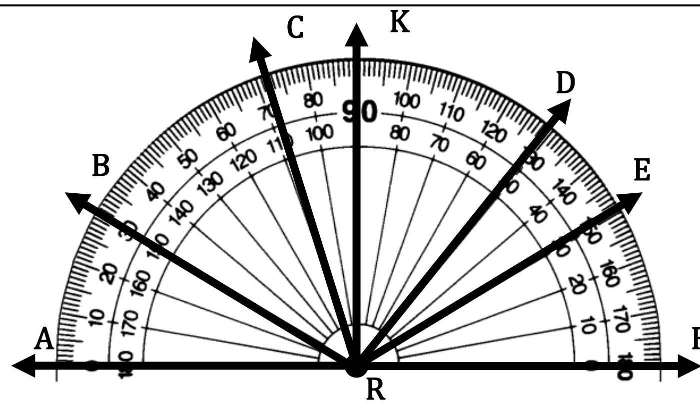
1) What is the measure of ∡DRA? ________

2) What is the measure of ∡CRF? ________

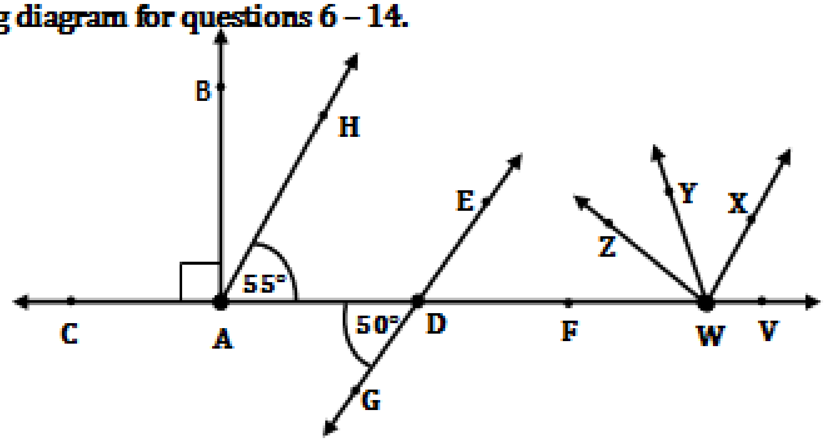
3) What is the measure of ∡ARB? ________

4) What is the measure of ∡CRB? _________

5) What is the measure of ∡KRC? _________



Use the following diagram for questions 6 – 14.



6) Which angle is supplementary angle to ∡EDF? ________

7) What is the measure of ∡GDF? ________

8) Which two angles are right angles? ______________ and ________________

9) What is the measure of ∡EDF? _________

10) Which angle is adjacent to ∡BAD? _________

11) Which angle is a complementary angle to ∡HAD? _________

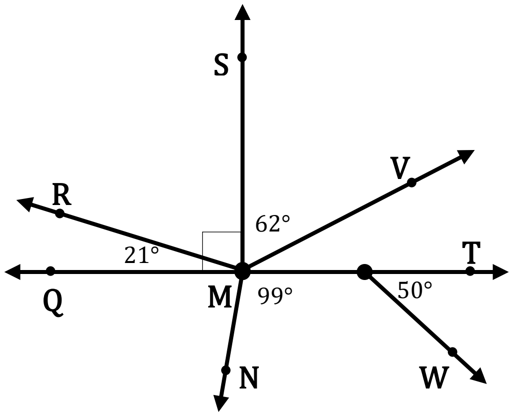
12) What is the measure of ∡HAB? ________

13) What is the measure of ∡CAD? ________

14) Which angles are adjacent to ∡EDA?

______________ and ________________

Part 2: Use what you know about complementary and supplementary angles to find the measures of the following angles.

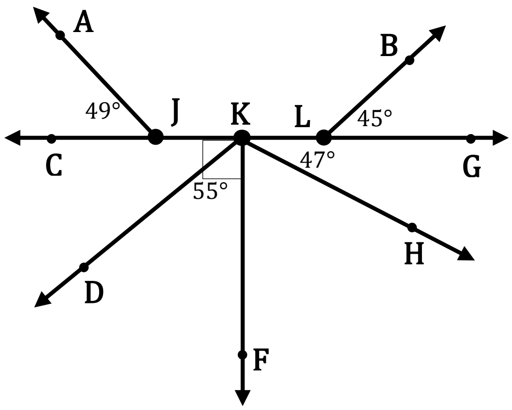


1) m∡ RMS = ____________

2) m∡ VMT = ____________

3)m∡ QMN = ____________

4) m∡ WPQ = ____________



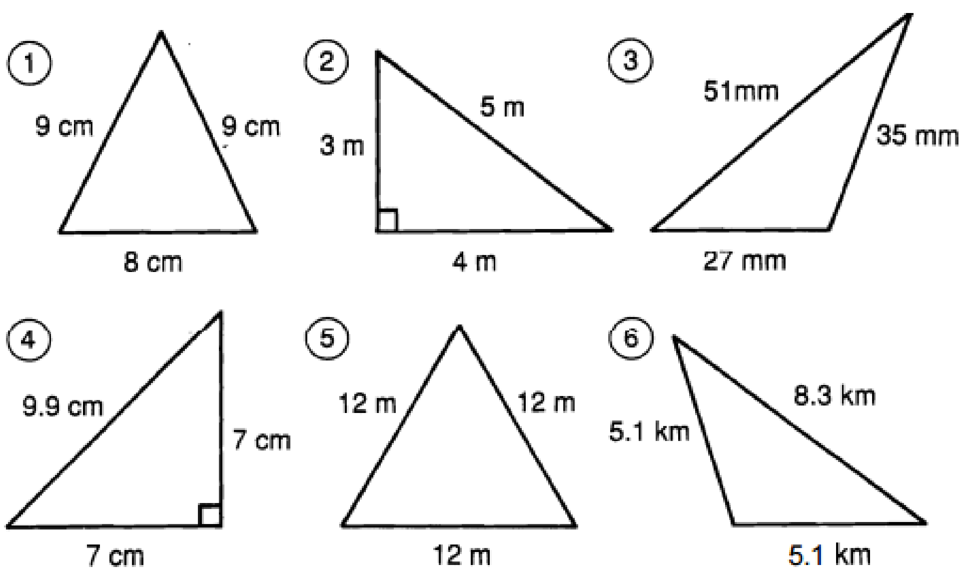
5) m∡ AJK = ____________

6) m∡ CKD = ____________

7)m∡ FKH = ____________

8) m∡ BLC = ____________

Part 3: Classify each triangle two ways.



1) ________________

____

2) ________________

____

3) ________________

____

4) ________________

____

5) ________________

____

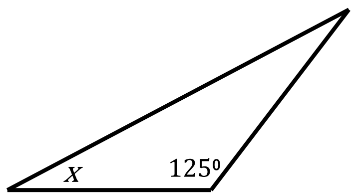
6) ________________

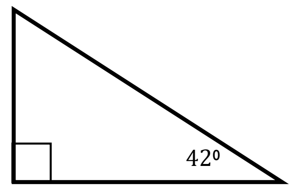
____

Interior Angles of a Triangle

FACT: The three interior angles of a triangle always add up to ___________^{0}.

Write an equation and solve to find the missing angle in the triangle.

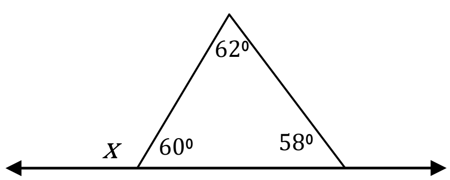




Exterior Angles of a Triangle

The exterior angle of a triangle is always equal to the sum of the opposite interior angles.

Example 1: Examine the figure below. Find the measure of the missing angle.



1) Sum ∡'s in triangle = _______

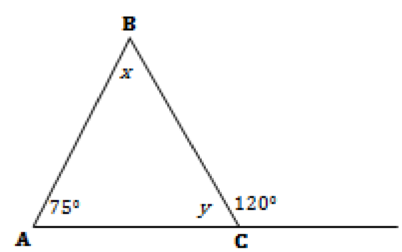
2) x = ___________________

3) Sum of interior angles opposite of angle "x"

= ______ + ______ = _________

Example 2: Find the measure of ∡x and ∡y.

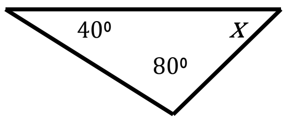
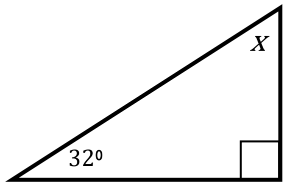
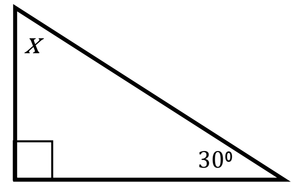
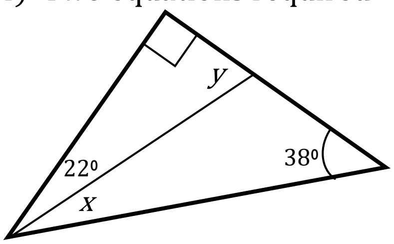
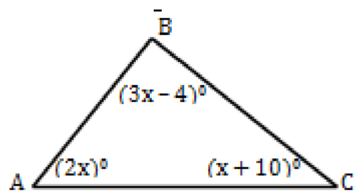
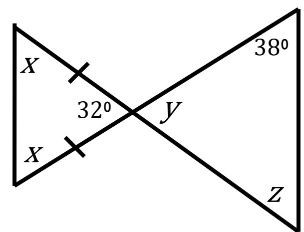
<u>Step 1:</u> Use the rule for exterior angles to write equation to solve for x.



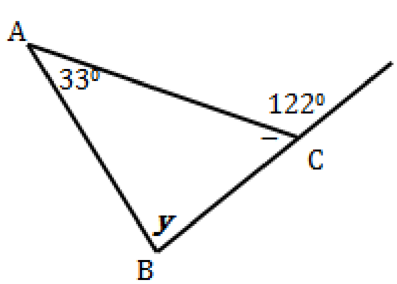
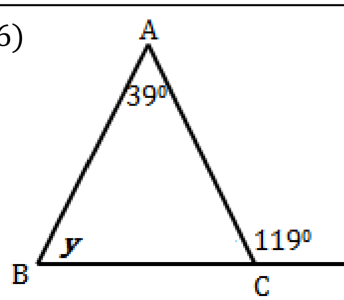
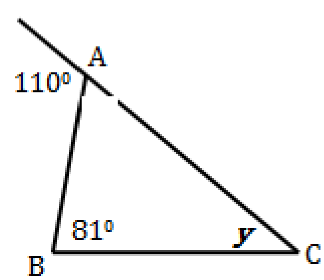
<u>Step 2:</u> The sum of the interior angles of a triangle equals 180^0, and ∡BCA supplements ∡BCD, so either

<u>SUM of INTERIOR ANGLES</u>	**<u>SUPPLEMENTAL ANGLES</u>**

Write an equation and solve to find the value of the variables.

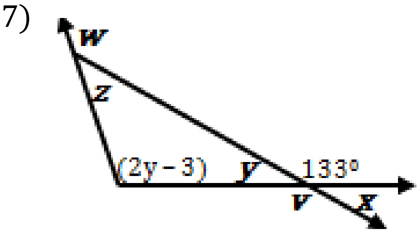
1) 	2) 	3) 
4) Two equations required  y: ________ z: ________ x: ________	5)  A: ________ B: ________ C: ________	6) Two equations required  x: ________ y: ________ z: ________

Write an equation using the sum of the interior angles and solve to find the value of the variables.

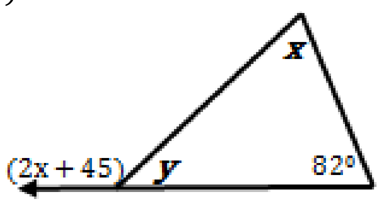
5) 	6) 	7) 

Write an equation and solve to find the value of the variables.

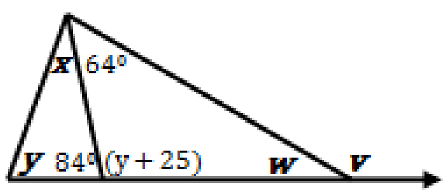
7)



8)

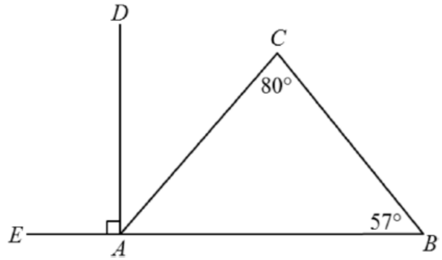


9)



The following problems are multiple choice. Circle the letter indicating the best answer for each question.

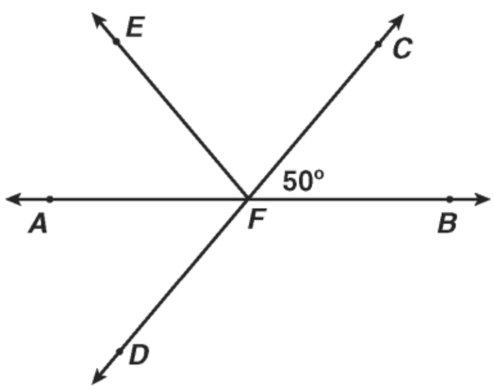
10) In the figure below, what is $m\angle DAC$?



A. 47° B. 57°

C. 90° D. 137°

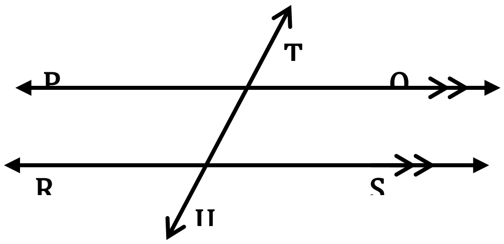
11) In the figure below, $\overleftrightarrow{CD}$ intersects $\overleftrightarrow{AB}$ at F, $m\angle CFB = 50°$, and $\angle EFA \cong \angle AFD$. What is $m\angle EFC$?



A. 40° B. 50° C. 70° D. 80°

Corresponding, Alternate Interior, and Alternate Exterior Angles

If two parallel lines are intersected by another line, how many angles are formed?



$\overline{PQ} \parallel \overline{RS}$

$\overline{TU}$ *is a transversal*

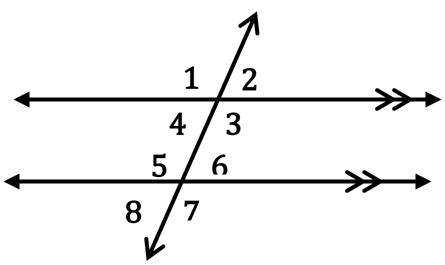
The extra arrows on two of the lines mean they are ____________________________.

The line that intersects the two lines is called a _______________________________.

The number of angles formed is ________.

The angles formed when parallel lines are cut by a transversal line have special relationships and are named according to those relationships with one another.

CORRESPONDING ANGLES



Definition:

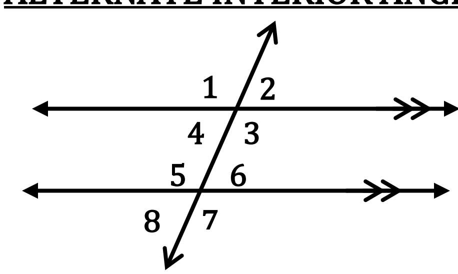
Name the corresponding angles for the following.

1) ∡1 corresponds with ∡_____
2) ∡2 corresponds with ∡_____
3) ∡3 corresponds with ∡_____
4) ∡4 corresponds with ∡_____

If two angles are ***corresponding*** angles,

then they are: __________________________________

ALTERNATE INTERIOR ANGLES



Definition:

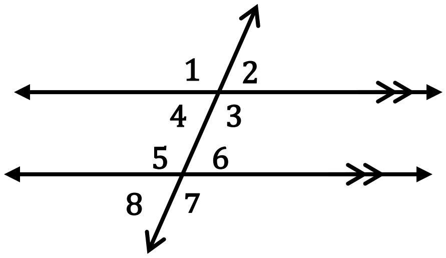
Name the alternate interior angle for the following angles.

1) ∡3 is an alternate interior angle with ∡_____
2) ∡4 is an alternate interior angle with ∡_____

If two angles are ***alternate interior*** angles,

then they are: ____

ALTERNATE EXTERIOR ANGLES



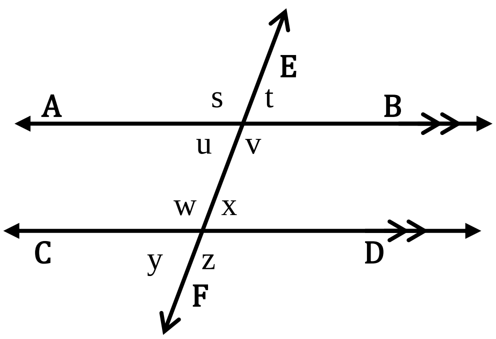
Definition:

Name the alternate exterior angle for the following angles.
1) ∡1 is an alternate exterior angle with ∡_____
2) ∡2 is an alternate exterior angle with ∡_____

If two angles are ***alternate exterior*** angles,

then they are: ___________________________________

Look at the diagram below. For each pair of angles, state whether they are corresponding (C), alternate interior (AI), alternate exterior (AE), vertical (V), or supplementary (S).



1) ∡u, ∡x ___________	6) ∡t, ∡x ___________
2) ∡w, ∡s ___________	7) ∡w, ∡z ___________
3) ∡t, ∡y ___________	8) ∡v, ∡w ___________
4) ∡s, ∡t ___________	9) ∡v, ∡z ___________
5) ∡w, ∡y ___________	10) ∡s, ∡z ___________

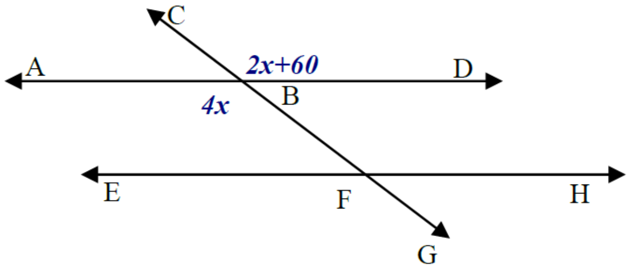
11) If m ∡s = 110º, write the measure of the remaining angles in the diagram.

Finding Unknown Angle Measures

We will use the angle relationships that are formed when two parallel lines are intersected by a transversal to find the measures of missing angles. All of the angle relationships will either be supplementary or congruent.

Example A: The pair of angles are either vertical angles, alternate interior angles, alternate exterior angles, or corresponding angles; so they are **congruent**. All you have to do is set up and solve an equation where the expressions are congruent. Once you have solved for x, substitute that value back into each expression to find the measure of each angle.

#1



Relationship: ______________________________

Equation: ________________________________

____

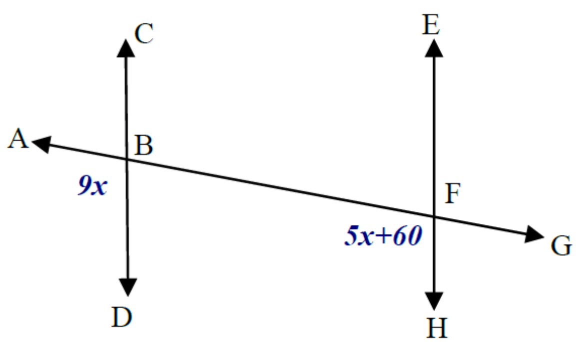
____

____

____

$x =$ ______ $\measuredangle ABG =$ _______ $\measuredangle CBD =$ _______

#2



Relationship: ______________________________

Equation: ________________________________

____

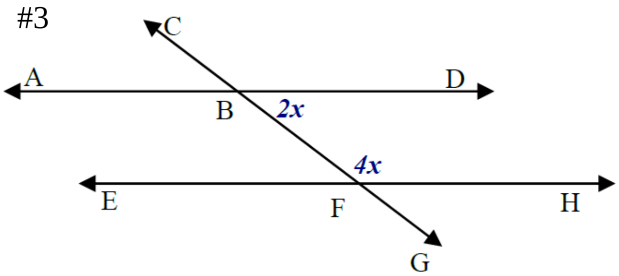
____

____

$x =$ ______ $\measuredangle ABD =$ _______ $\measuredangle HFA =$ _______

Example B: Each pair of angles are **supplementary** to each other, which means the angles add up to 180°. All you have to do is set up and solve an equation where the expressions add up to equal 180°. Once you have solved for x, substitute that value back into each expression to find the measure of each angle.

#3



Relationship: ______________________________

Equation: ________________________________

____

____

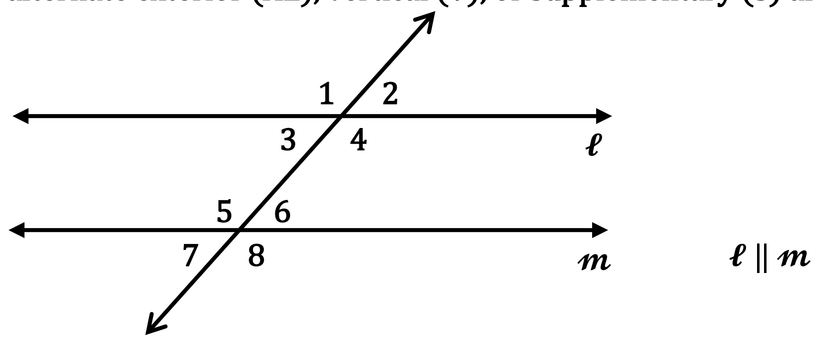
____

____

$x =$ ______ $\measuredangle FBD =$ _______ $\measuredangle HFB =$ _______

Independent Practice:

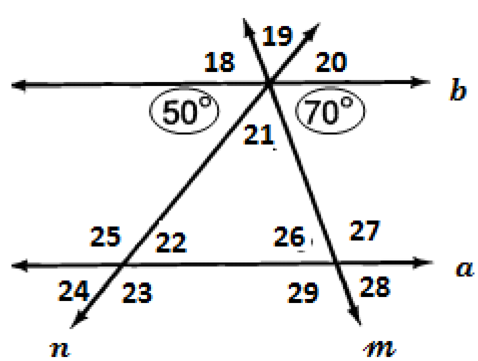
Part 1: For each pair of angles, state whether they are corresponding (C), alternate interior (AI), alternate exterior (AE), vertical (V), or supplementary (S) angles.



1) ∡1 and ∡4 ________	6) ∡6 and ∡5 ________
2) ∡2 and ∡6 ________	7) ∡2 and ∡7 ________
3) ∡1 and ∡3 ________	8) ∡1 and ∡2 ________
4) ∡5 and ∡8 ________	9) ∡4 and ∡5 ________

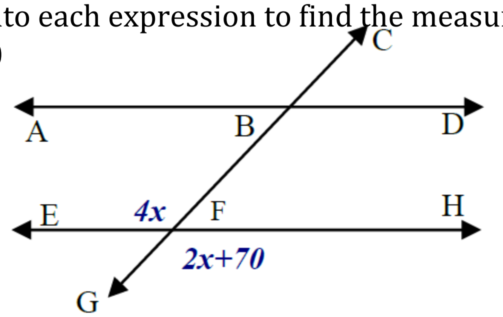
Part 2: Parallel lines *a* and *b* when cut by transversals *m* and *n*. Find all of the unknown angle measures.

18) m ∡18 = ________	19) m ∡19 = ________
20) m ∡20 = ________	21) m ∡21 = ________
22) m ∡22 = ________	23) m ∡23 = ________
24) m ∡24 = ________	25) m ∡25 = ________
26) m ∡26 = ________	27) m ∡27 = ________
28) m ∡28 = ________	29) m ∡29 = ________



<u>Part 1:</u> Find the measure of each missing angle in the parallel lines and transversals. Each pair of angles is either <u>supplementary</u> or congruent (<u>vertical angles</u>, <u>alternate interior angles</u>, <u>alternate exterior angles</u>, or <u>corresponding angles</u>). State the relationship, set up an appropriate equation and solve for x. Once you've solved for x, substitute that value back into each expression to find the measure of each angle.

1)



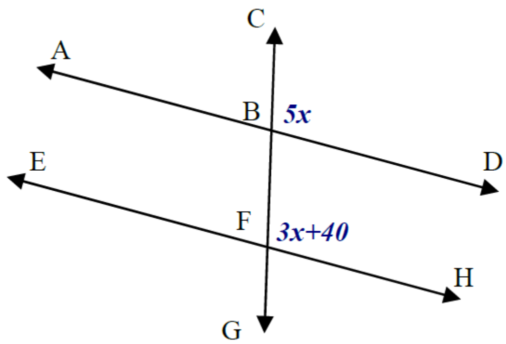
Relationship: ____________________

Equation: ____________________

Solve:

$x =$ ______ $\measuredangle EFB =$ ________ $\measuredangle GFH =$ ________

2)



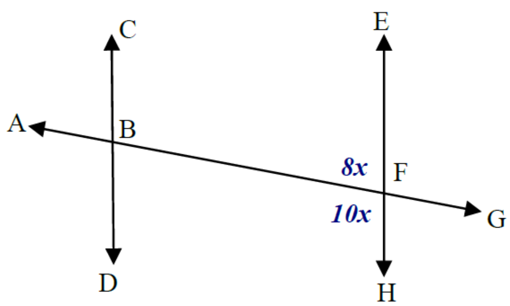
Relationship: ____________________

Equation: ____________________

Solve:

$x =$ ______ $\measuredangle CBD =$ ________ $\measuredangle BFH =$ ________

3)



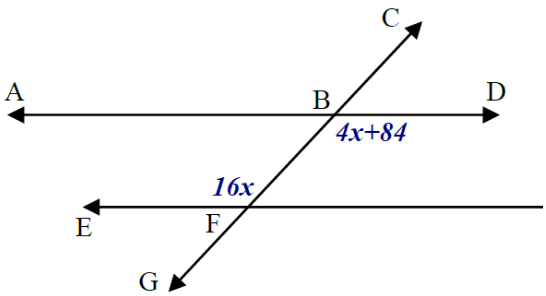
Relationship: ____________________

Equation: ____________________

Solve:

$x =$ ______ $\measuredangle EFB =$ ________ $\measuredangle BFH =$ ________

4)



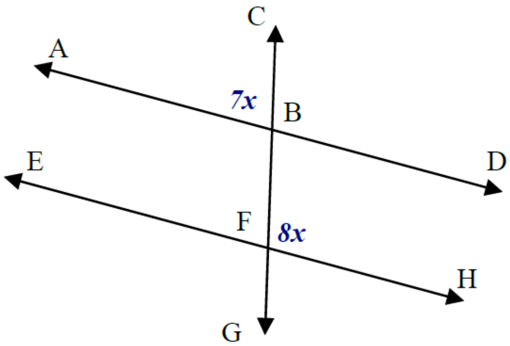
Relationship: ____________________

Equation: ____________________

Solve:

$x =$ ______ $\measuredangle EFB =$ ________ $\measuredangle DBF =$ ________

5)



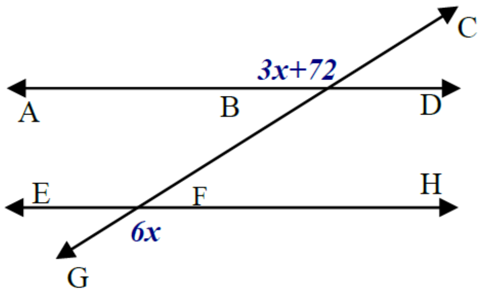
Relationship: ______________________________

Equation: ______________________________

Solve:

$x =$ _______ $\measuredangle CBA =$ _________ $\measuredangle BFH =$ _________

6)



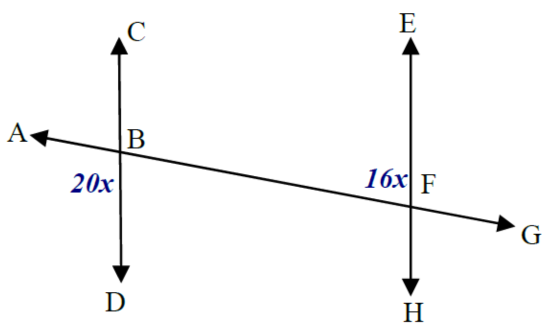
Relationship: ______________________________

Equation: ______________________________

Solve:

$x =$ _______ $\measuredangle CBA =$ _________ $\measuredangle GFH =$ _________

7)



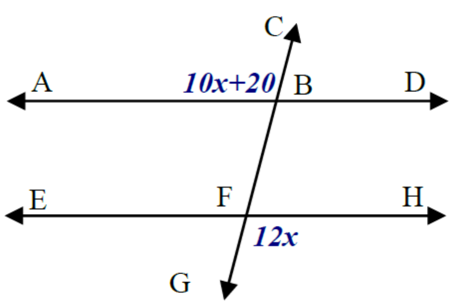
Relationship: ______________________________

Equation: ______________________________

Solve:

$x =$ _______ $\measuredangle DBA =$ _________ $\measuredangle EFB =$ _________

8)



Relationship: ______________________________

Equation: ______________________________

Solve:

$x =$ _______ $\measuredangle CBA =$ _________ $\measuredangle GFH =$ _________