

[70] Squares of Binomials

Multiply.

$$\textcircled{1} (x+3)^2 = \overbrace{(x+3)(x+3)} \\ \boxed{x^2 + 6x + 9}$$

$$\textcircled{2} (x-6)^2 = \overbrace{(x-6)(x-6)} \\ \boxed{x^2 - 12x + 36}$$

$$\textcircled{3} (5x+3)^2 = \overbrace{(5x+3)(5x+3)} \\ \boxed{25x^2 + 30x + 9}$$

Factor Perfect Square Trinomials

$$\textcircled{4} x^2 - 8x + 16 \quad \begin{array}{l} \swarrow \text{Always} \\ \text{addition} \end{array} \\ (x-4)(x-4) \text{ or } (x-4)^2$$

$$\textcircled{5} 9x^2 - 6x + 1 \\ (3x-1)(3x-1) \text{ or } (3x-1)^2$$

$$\textcircled{6} 49x^2 - 28x + 4 \\ (7x-2)(7x-2) \text{ or } (7x-2)^2$$

$$\textcircled{7} 9x^2 + 30x + 100 \\ (3x+10)(3x+10) \quad \boxed{\text{prime}} \quad \begin{array}{l} \text{Not} \\ \text{Factorable} \end{array}$$

$$\textcircled{8} 8x^2 + 8x + 2 \\ 2(4x^2 + 4x + 1) \\ 2(2x+1)(x+1) \text{ or } 2(2x+1)^2$$

NAME _____

DATE _____

7D Squares of Binomials

Perfect Square Trinomial

Objective: To find squares of binomials and to factor perfect square trinomials.**Vocabulary****Square of a Binomial**

$$(a + b)^2 = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

$$(a - b)^2 = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2$$

Perfect Square Trinomial

$$a^2 + 2ab + b^2 = (a + b)^2$$

$$a^2 - 2ab + b^2 = (a - b)^2$$

Example 1 Write each square as a trinomial.

a. $(x + 4)^2$ b. $(5u - 2)^2$ c. $(2x + 3y)^2$ d. $(5a^2 - 4b^2)^2$

Solution Use the patterns for the square of a binomial.

a. $(x + 4)^2 = x^2 + 2(x \cdot 4) + 4^2$ Pattern: $(a + b)^2 = a^2 + 2ab + b^2$
 $= x^2 + 8x + 16$

b. $(5u - 2)^2 = (5u)^2 - 2(5u \cdot 2) + 2^2$ Pattern: $(a - b)^2 = a^2 - 2ab + b^2$
 $= 25u^2 - 20u + 4$

c. $(2x + 3y)^2 = (2x)^2 + 2(2x \cdot 3y) + (3y)^2$ Pattern: $(a + b)^2 = a^2 + 2ab + b^2$
 $= 4x^2 + 12xy + 9y^2$

d. $(5a^2 - 4b^2)^2 = (5a^2)^2 - 2(5a^2 \cdot 4b^2) + (4b^2)^2$ Pattern:
 $= 25a^4 - 40a^2b^2 + 16b^4$ $(a - b)^2 = a^2 - 2ab + b^2$

Write each square as a trinomial

1. $(x + 3)^2$
3. $(a - 4)^2$
5. $(3u - 1)^2$
7. $(3y + 4)^2$
9. $(5k - 2)^2$
11. $(3p + 5q)^2$
13. $(3y - 5)^2$
15. $(2x + y)^2$
17. $(ef - 7)^2$
19. $(2ab - c)^2$
21. $(-4 + 3f)^2$
23. $(5p^3 - 6)^2$

Evens

2. $(y - 2)^2$
4. $(n + 5)^2$
6. $(5c - 1)^2$
8. $(4k + 5)^2$
10. $(ab - 3)^2$
12. $(4x - 3y)^2$
14. $(5a - 7b)^2$
16. $(x^2 + 5)^2$
18. $(pq - 3)^2$
20. $(-3ab + b^2)^2$
22. $(-11u + v^2)^2$
24. $(-9r^2 - 2)^2$

7D Squares of Binomials (continued)

Example 2 Decide whether each trinomial is a perfect square. If it is, factor it.

a. $4x^2 - 12x + 9$ b. $16u^2 + 20uv + 25v^2$

Solution a. $4x^2 - 12x + 9$

1. Is the first term a square?

Yes; $4x^2 = (2x)^2$

2. Is the last term a square?

Yes; $9 = (3)^2$

3. Is the middle term, neglecting the sign, twice the product of $2x$ and 3 ?

Yes; $12x = 2(2x \cdot 3)$

Since the answers to Questions 1-3 are all Yes,

$4x^2 - 12x + 9$ is a perfect square.

Use the pattern $a^2 - 2ab + b^2 = (a - b)^2$:
 $4x^2 - 12x + 9 = (2x - 3)^2$.

b. $16u^2 + 20uv + 25v^2$

1. Is the first term a square?

Yes; $16u^2 = (4u)^2$

2. Is the last term a square?

Yes; $25v^2 = (5v)^2$

3. Is the middle term, neglecting the sign, twice the product of $4u$ and $5v$?

No; $20uv \neq 2(4u \cdot 5v)$

Since the answer to Question 3 is No,

$16u^2 + 20uv + 25v^2$ is not a perfect square.

Decide whether each trinomial is a perfect square. If it is, factor it. If it is not, write *not a perfect square*. (Prime)

25. $n^2 - 4n + 4$

26. $k^2 + 10k + 25$

27. $a^2 - 6a + 9$

28. $y^2 - 8y + 16$

29. $a^2 - 4a + 16$

30. $81 + 18y + y^2$

31. $9x^2 - 12x + 4$

32. $16k^2 - 40k + 25$

33. $9y^2 + 48y + 64$

34. $9 + 6y + 4y^2$

35. $25x^2 + 80xy + 64y^2$

36. $4c^2 - 12c + 9$

37. $9n^2 - 24n + 16$

38. $81 - 36k + 4k^2$

39. $81k^2 + 180k + 100$

40. $49a^2 - 42ab + 9b^2$

41. $4m^2 - 36mn + 81n^2$

42. $16x^2 - 24xy + 9y^2$

Mixed Review Exercises

Evaluate if $x = 3$ and $y = 2$.

1. $x + y + (-6)$

2. $x - |5 - y|$

3. $6 + x^2y$

4. $(3 + xy)^2$

5. $(x)^3(-y)^3$

6. $(x^2y^2)^2$

7-12 Simplify.

7. $(2x + 5)(2x - 5)$

8. $(x - 6)(x + 2)$

9. $(6 - 2)^3$

10. $4 - 2^3$

11. $\frac{(a^4)^3}{(a^2)^4}$

12. $\frac{(3xy)^2}{9xy}$