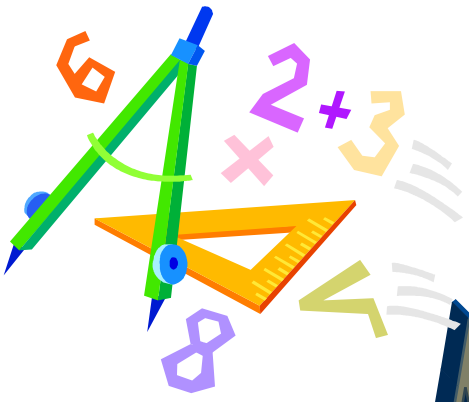
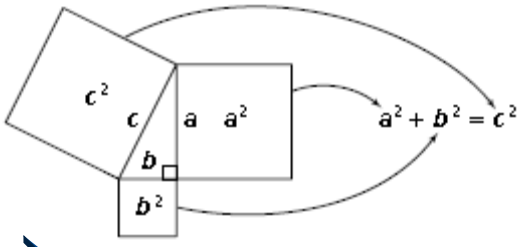
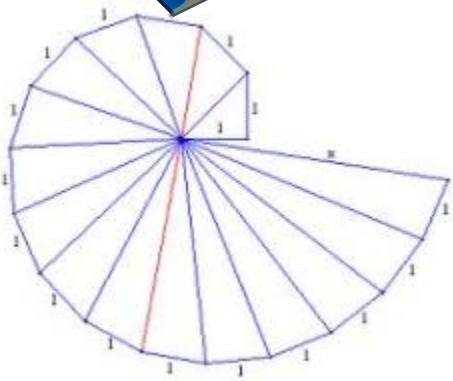
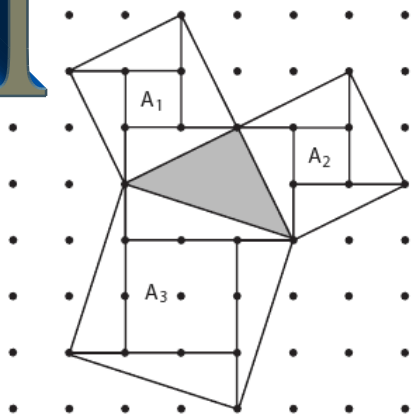


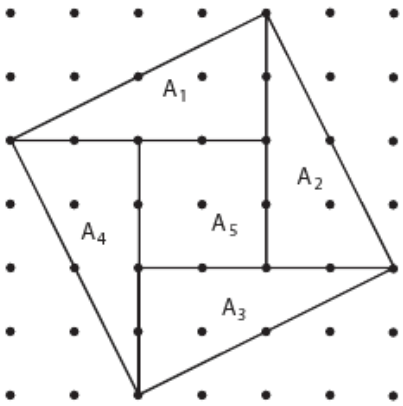
Name: _____



IRRATIONAL MATH



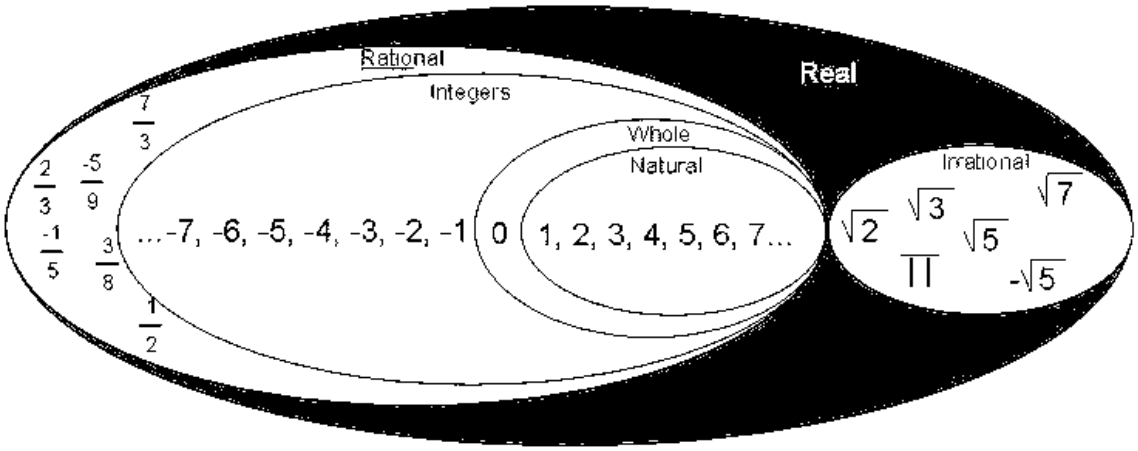




2015-2016 **Advanced Algebra I**

Notes

Real Number System



Definition: A rational number is a number that can be expressed as a fraction (ratio) in the form $\frac{p}{q}$ where p and q are integers and q is not zero.

Examples: $\frac{1}{2},\ 8,\ \frac{5}{3},\ \sqrt{4},\ 7\frac{1}{9},\ -12,\ \frac{\sqrt{64}}{\sqrt{25}},\ 6.25,\ 0.3\overline{18}$

A rational number can be expressed as a ratio (fraction).

When a rational number fraction is divided to form a decimal value, it becomes a **terminating** or **repeating decimal**.

$\frac{3}{4}$ can be represented as $4\overline{)3.00}^{\,0.75}$ which is a terminating decimal.

$\frac{2}{3}$ can be represented as $3\overline{)2.0}^{\,0.\overline{6}}$ which is a repeating decimal.

Definition: An irrational number is a number that is NOT rational. It cannot be expressed as a fraction with integer values in the numerator and denominator.

Examples: $\frac{\sqrt{3}}{2},\ \pi,\ -\sqrt{27},\ 0.131331333...,\ \frac{\sqrt{13}}{\sqrt{2}},\ 4\pi,\ 3+\sqrt{5}$

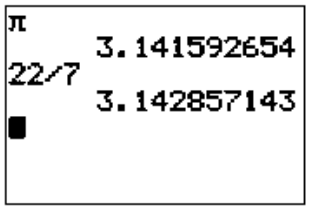
When an irrational number is expressed in decimal form, it **goes on forever without repeating.**

Regarding π:

$\pi \neq \frac{22}{7}$

$\pi \neq 3.14$

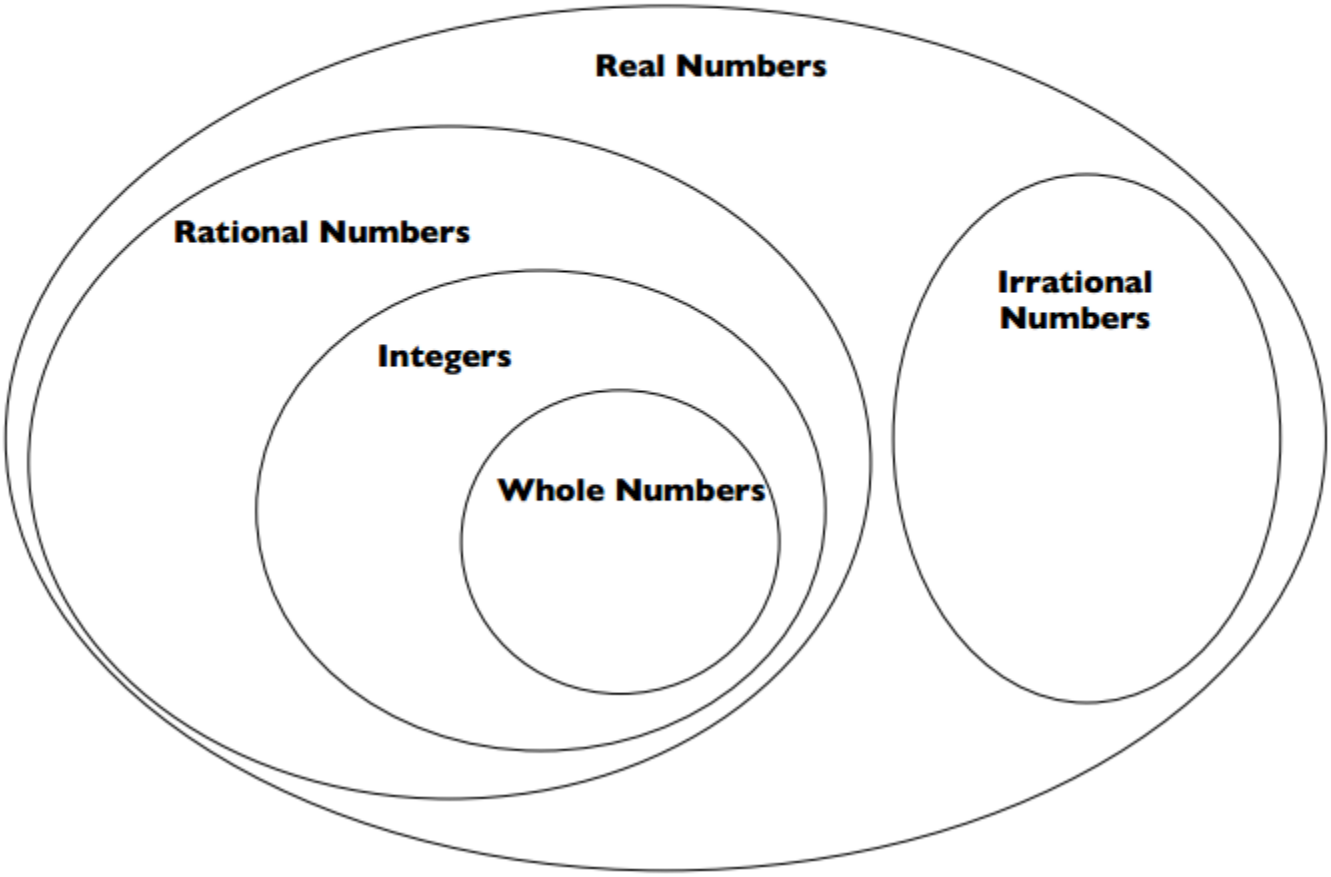
While it is popular to use 3.14 or $\frac{22}{7}$ to represent "pi", these values are only estimates or approximations. Notice the differences in the decimal representations on the calculator screen at the right.



$\pi = 3.14159265358979323846264338327950288419716939937510582097...$

Write each number in the correct location on the Venn Diagram of the real number system. Each number should be written only once.

$$\left\{-6,\ 2.73,\ \frac{3}{7},\ \sqrt{2},\ \sqrt{9},\ -100,\ 0,\ \pi,\ 1,\ -\frac{1}{2},\ -3.8,\ 5.\overline{42},\ 8.293017\ldots\right\}$$



Put a check mark for **each set** that the number is a part of:

	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
-7					
$\frac{3}{4}$					
$\sqrt{2}$					
5					
0.398					

Creating Squares

Notes:

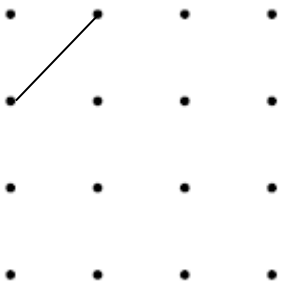
Measure each of the following segments to the nearest tenth of a cm.

	Segment	***Measure***
1.	_____	_____cm
2.	_____	_____cm
3.	_____	_____cm
4.	_____	_____cm

State the attributes of a square: _____

State the slope and the perpendicular slope for each line segment. Draw a square using the segment as one side length.

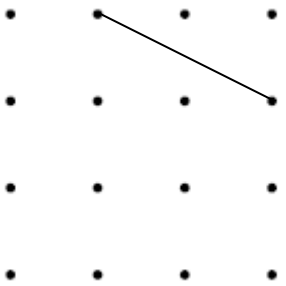
5.



Slope: _____

⊥ Slope: _____

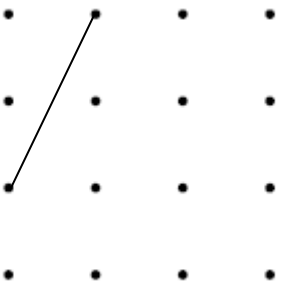
6.



Slope: _____

⊥ Slope: _____

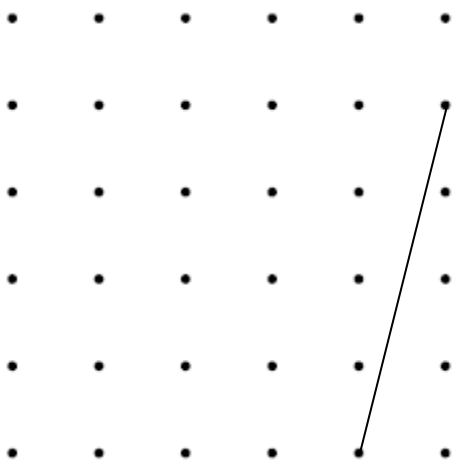
7.



Slope: _____

⊥ Slope: _____

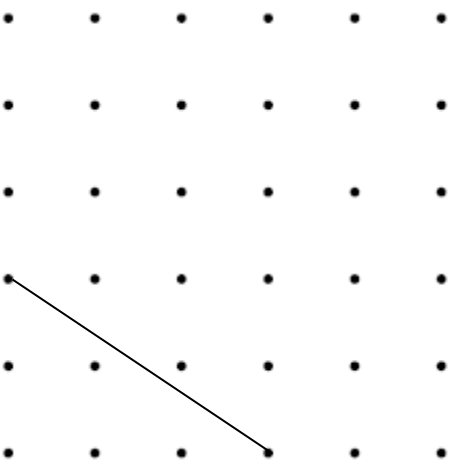
8.



Slope: _____

⊥ Slope: _____

9.



Slope: _____

⊥ Slope: _____

Assignment. Part 1

The Number System

Identify the sets to which each of the following numbers belongs by marking an "X" in the appropriate boxes.

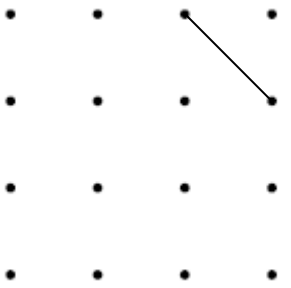
	Number	Natural Numbers	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
1.	$-\sqrt{17}$						
2.	-2						
3.	$-\frac{9}{37}$						
4.	0						
5.	-6.06						
6.	$4.5\overline{6}$						
7.	3.050050005...						
8.	18						
9.	$\sqrt{50}$						
10.	π						
11.	$\overline{.634}$						
12.	$\sqrt{225}$						
13.	.634						
14.	$\sqrt{\frac{4}{49}}$						
15.	$-\sqrt{64}$						

Assignment. Part 2

Measure each of the following segments to the nearest tenth of a cm.

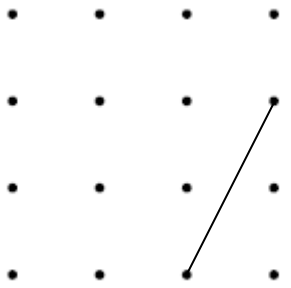
	Segment	***Measure***
1.	_____	_____cm
2.	_____	_____cm
3.	_____	_____cm
4.	_____	_____cm

State the slope and the perpendicular slope for each line segment. Draw a square using the segment as one side length.

5. 

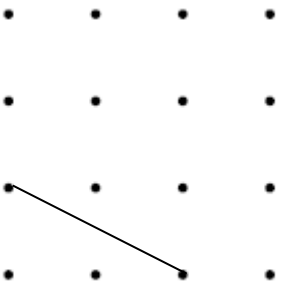
Slope: _____

⊥ Slope: _____

6. 

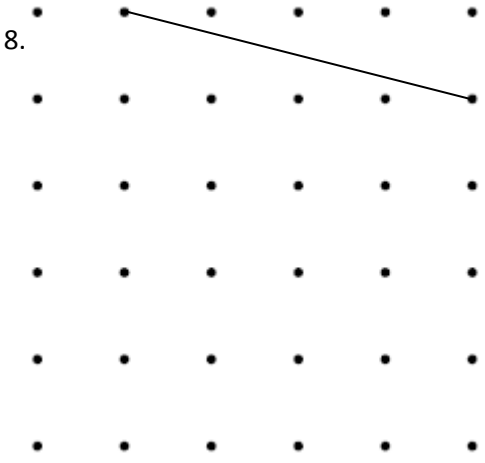
Slope: _____

⊥ Slope: _____

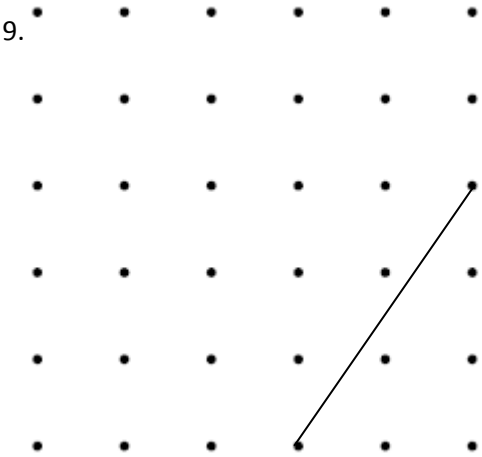
7. 

Slope: _____

⊥ Slope: _____

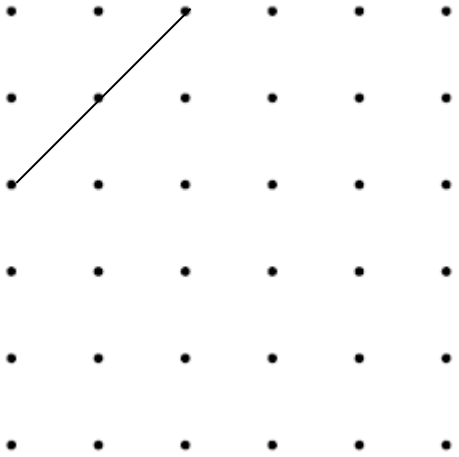
8. 

Slope: _____ ⊥ Slope: _____

9. 

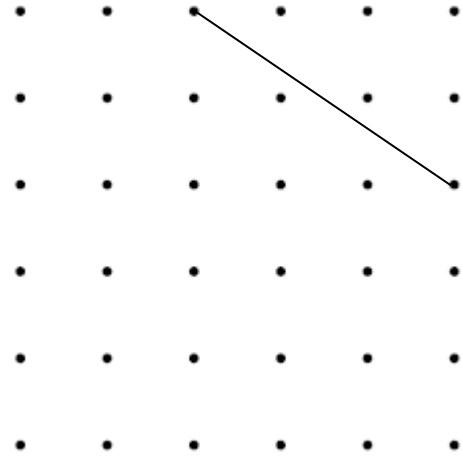
Slope: _____ ⊥ Slope: _____

10.



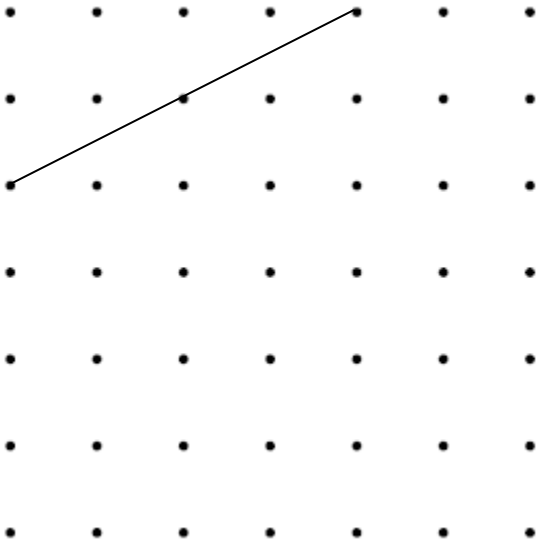
Slope: _____ ⊥ Slope: _____

11.



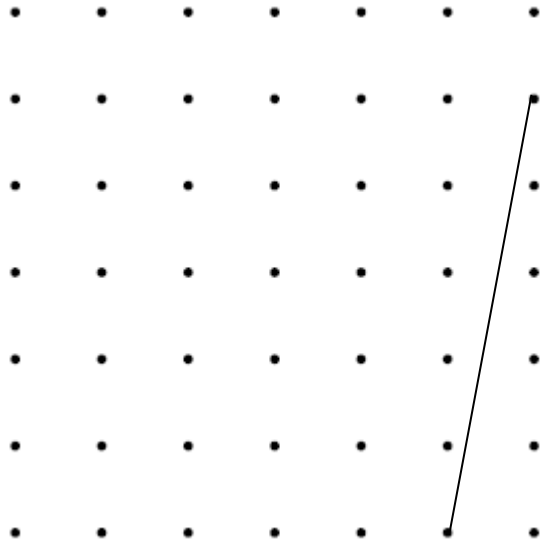
Slope: _____ ⊥ Slope: _____

12.



Slope: _____ ⊥ Slope: _____

13.



Slope: _____ ⊥ Slope: _____

Assignment. Part 3

Multiple Choice

1. Which set below includes only irrational numbers?

A. $\left\{-\sqrt{12},\ -3.7\overline{6},\ \sqrt{36},\ 4.3858\ldots\right\}$

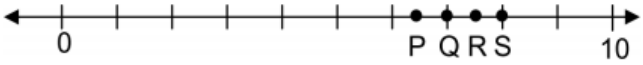
B. $\{-7.2322\ldots,\ \sqrt{5},\ \sqrt{15},\ 8.27451\ldots\}$

C. $\left\{-5.6,\ \sqrt{14},\ 6.3\overline{245},\ \sqrt{81}\right\}$

D. $\left\{-\sqrt{8},\ .3\overline{7},\ 3.265165065\ldots,\ \sqrt{90}\right\}$

2. Which point on the number line shows the *best* estimate of the irrational number below?

$\sqrt{57}$



A. P B. Q C. R D. S

Estimations of Irrational Numbers

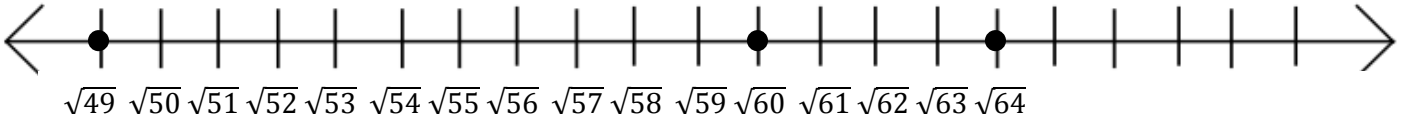
Notes:

For the following assignment, do NOT use a calculator.

Example: Approximate $\sqrt{60}$

What two consecutive perfect squares is 60 in between? *49 and 64*

The $\sqrt{49}$ =7 and $\sqrt{64}$ =8. So the $\sqrt{60}$ is between 7 and 8.

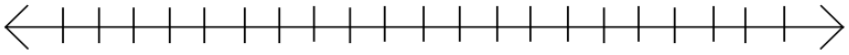


Since 60 is closer to 64, $\sqrt{60}$ will be closer to the 8. You might estimate 7.7 or 7.8. (If you use a calculator, you will find that $\sqrt{60} \approx 7.74597$) That is a pretty close estimation.

_____ < $\sqrt{60}$ < _____

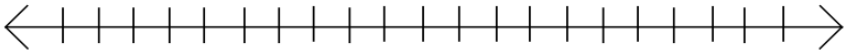
_____ < $\sqrt{60}$ < _____ so I approximate: _____

Approximate the following to the nearest tenth:



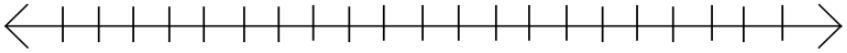
1) _____ < $\sqrt{45}$ < _____

_____ < $\sqrt{45}$ < _____ so I approximate: _____



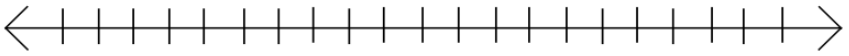
2) _____ < $\sqrt{24}$ < _____

_____ < $\sqrt{24}$ < _____ so I approximate: _____



3) _____ < $\sqrt{6}$ < _____

_____ < $\sqrt{6}$ < _____ so I approximate: _____



4) _____ < $\sqrt{66}$ < _____

_____ < $\sqrt{66}$ < _____ so I approximate: _____

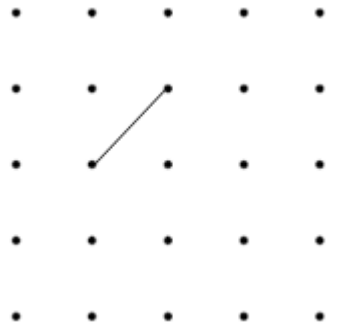
Estimate the following square roots to the nearest tenth. You must show the two perfect squares that the number is between. NO CALCULATOR!!!

5) $\sqrt{38}$ 6) $\sqrt{95}$ 7) $\sqrt{31}$ 8) $\sqrt{128}$

Finding the Areas of Squares Using Right Triangles & Rectangles

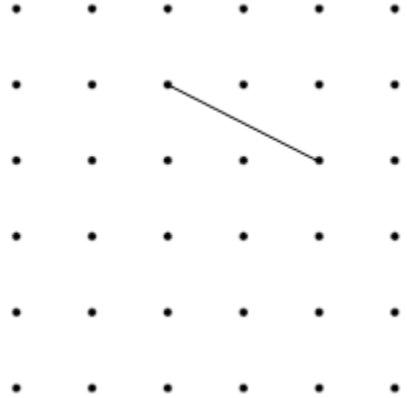
For each segment, draw a square, then use right triangles and rectangles to find the exact area of the tilted square. You must show your work as demonstrated.

1.



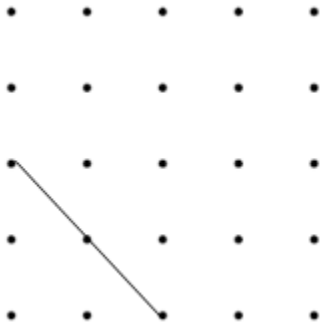
Area of square =_____

2.



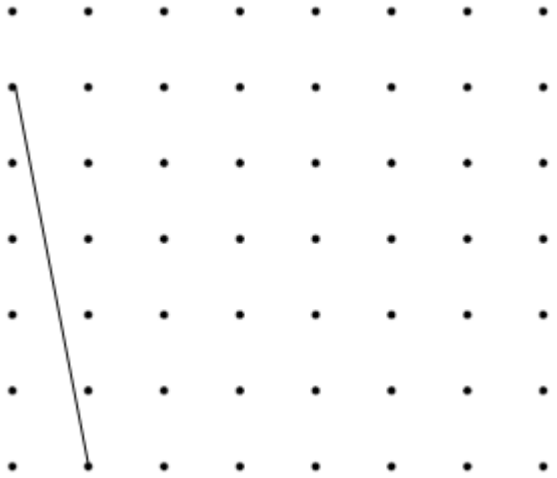
Area of square =_____

3.

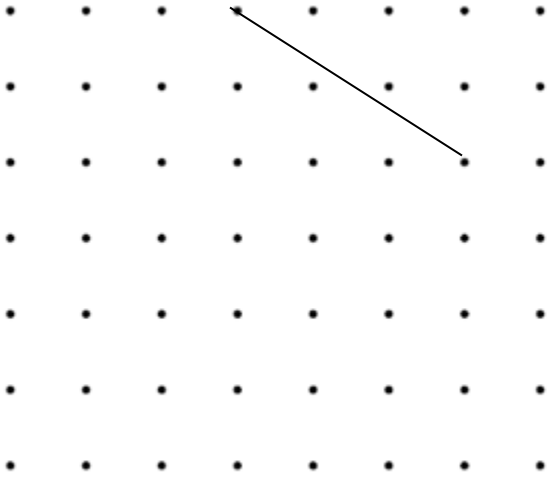


Area of square =_____

4.



Area of square =_____



Area of square =_____

Assignment, Part 1

Estimate the following square roots to the nearest tenth. You **must** show the two perfect squares that the number is between. NO CALCULATOR!!!

1) $\sqrt{95}$

2) $\sqrt{19}$

3) $\sqrt{390}$

4) $\sqrt{150}$

5) $\sqrt{45}$

6) $\sqrt{200}$

7) $\sqrt{119}$

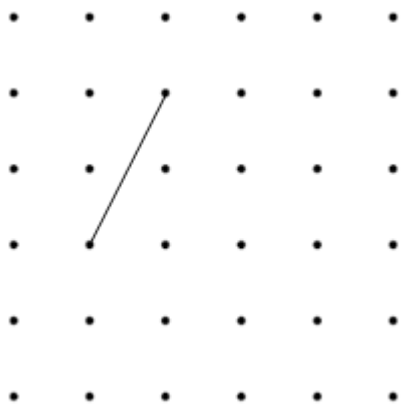
8) $\sqrt{251}$

9) $\sqrt{62}$

Assignment, Part 2

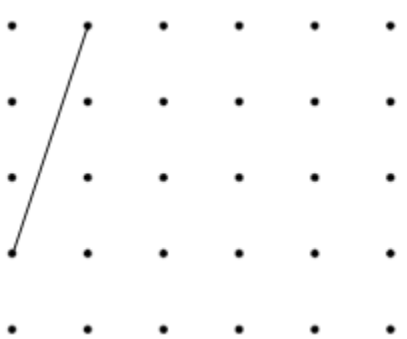
For each segment, draw a square, then use right triangles and rectangles to find the exact area of the tilted square. You must show your work as demonstrated.

1.



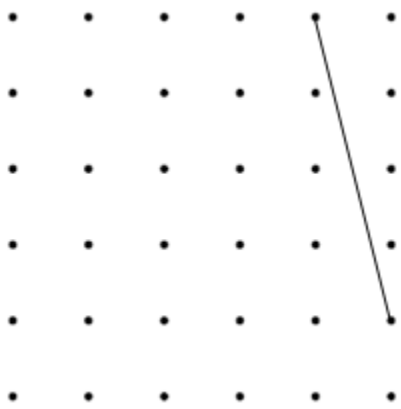
Area of square =_____

2.



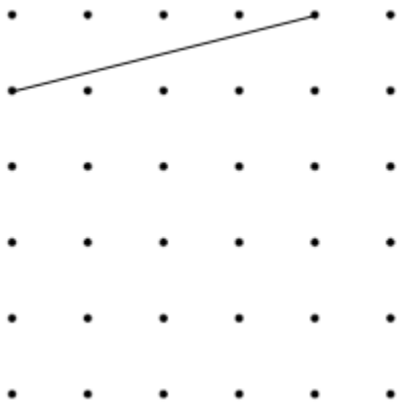
Area of square =_____

3.



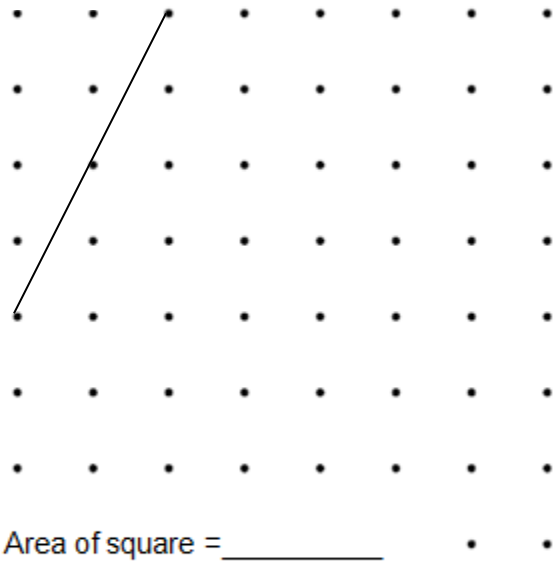
Area of square =_____

4.



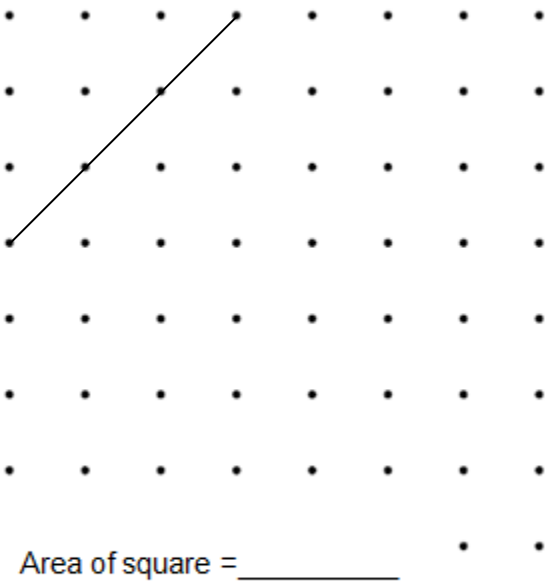
Area of square =_____

5.



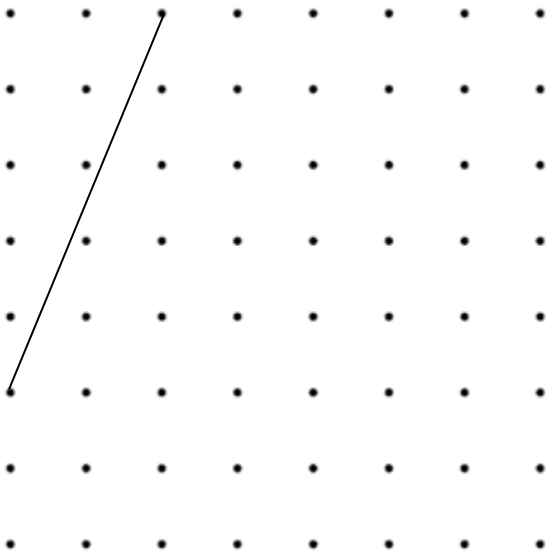
Area of square =_____

6.



Area of square =_____

7.



Area of square =_____

Assignment, Part 3

Identify the sets to which each of the following numbers belongs by marking and "X" in the appropriate boxes.

Number	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
-0.5					
-4					
$1.\overline{6}$					
$\sqrt{1}$					
8					
$\sqrt{5}$					
2.6469…					
$-2.\overline{45}$					
0					

Assignment, Part 4

Multiple Choice

1. Which set contains an irrational number?

   A. $\{2300,\ 0.48,\ \frac{13}{1}\}$

   B. $\{18,\ 0.1,\ \frac{12}{5}\}$

   C. $\{\frac{3}{8},\ 4,\ \sqrt{52}\}$

   D. $\{0.333\ldots,\ \sqrt{4},\ 10\}$

2. What type of number is $\sqrt{26}$?

   A. Whole number B. Integer

   C. Rational number D. Irrational number

3. Which number is irrational?

   A. $(1.5)^2$ B. $\sqrt{41}$ C. $\sqrt{49}$ D. $(15)^2$

Finding the Length of Segments

Given the area of the following squares, find the length of each side.

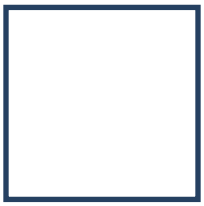
s = 2 cm

Area: $\underline{\mathbf{4\ cm^2}}$

Side: $\underline{\sqrt{4} = 2cm}$

s = 2 cm

s = _____

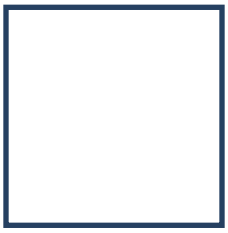


Area: $\underline{\mathbf{9\ cm^2}}$

Side: _____

s = _____

s = _____

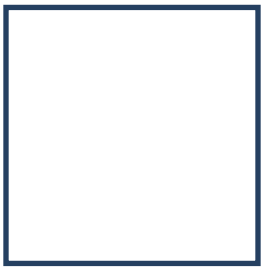


Area: $\underline{\mathbf{81\ cm^2}}$

Side: _____

s = _____

s = _____

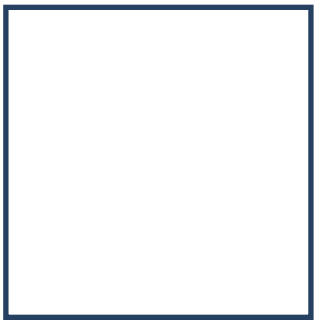


Area: $\underline{\mathbf{225\ cm^2}}$

Side: _____

s = _____

s = _____

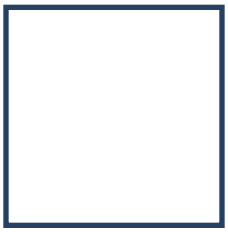


Area: $\underline{\mathbf{841\ cm^2}}$

Side: _____

s = _____

s = _____



Area: $\underline{\mathbf{8\ cm^2}}$

Side: _____

s = _____

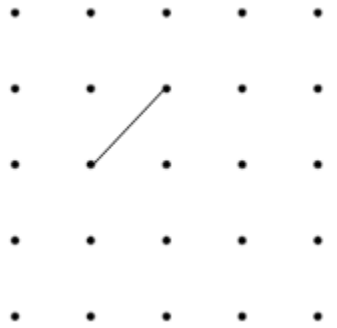
When you know the area of a square, you can take the square root of the area to get the side length of the square.

Given a square's area, $\sqrt{area} = side\ length$

Finding Lengths of Segments with Irrational Measurements

Notes: Draw a Square. Find the area of the square. Use the area to find the length of the side. Estimate your answer by measuring with a ruler and using your calculator.

1)



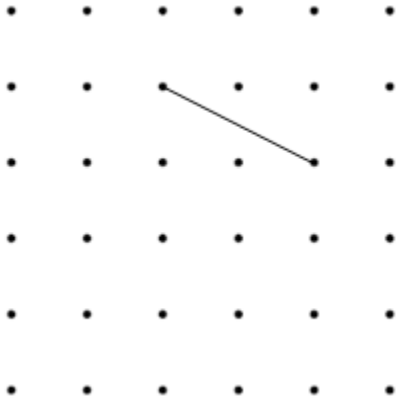
Area of square =_____

Length of the segment = $\sqrt{}$ _____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

2)



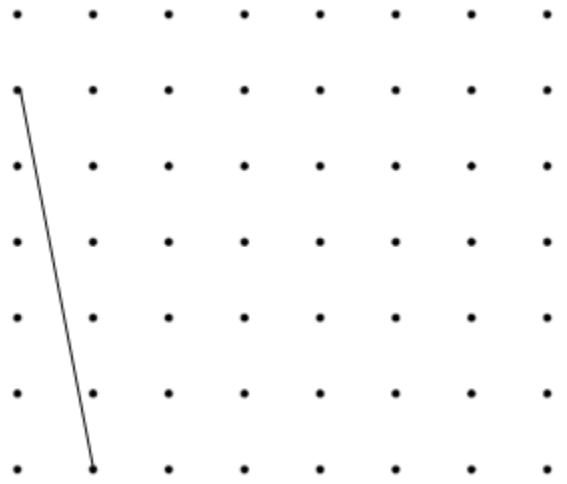
Area of square =_____

Length of the segment = $\sqrt{}$ _____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

3)



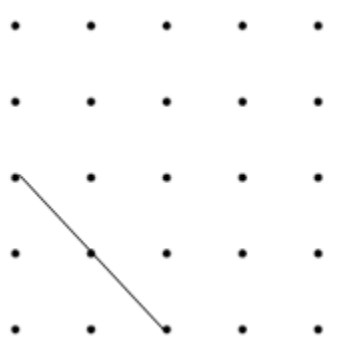
Area of square =_____

Length of the segment = $\sqrt{}$ _____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

4)



Area of square =_____

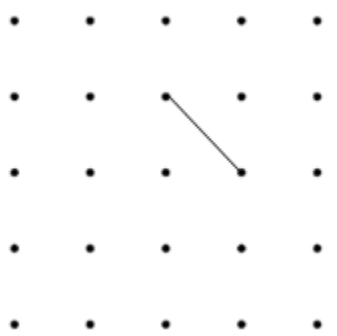
Length of the segment = $\sqrt{}$ _____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

Assignment, Part 1

Draw a Square. Find the area of the square. Use the area to find the length of the side. Estimate your answer by measuring with a ruler and using your calculator.

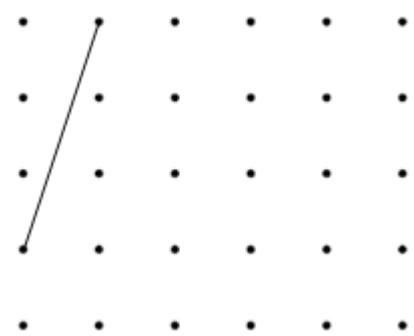
1) 

Area of square =_____

Length of the segment = $\sqrt{}$_____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

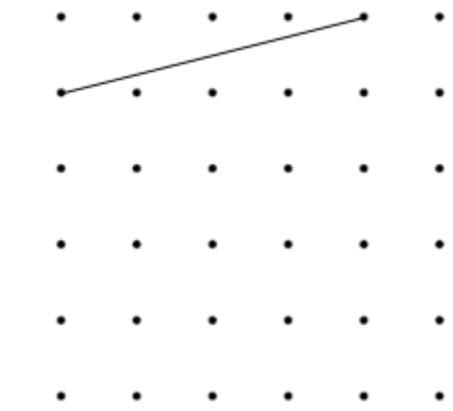
2) 

Area of square =_____

Length of the segment = $\sqrt{}$_____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

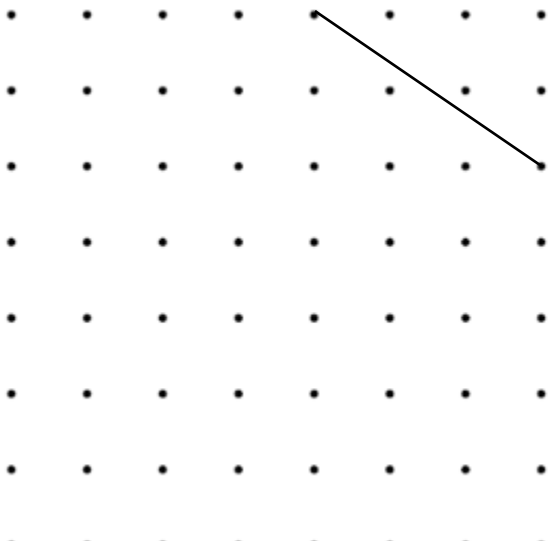
3) 

Area of square =_____

Length of the segment = $\sqrt{}$_____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

4) 

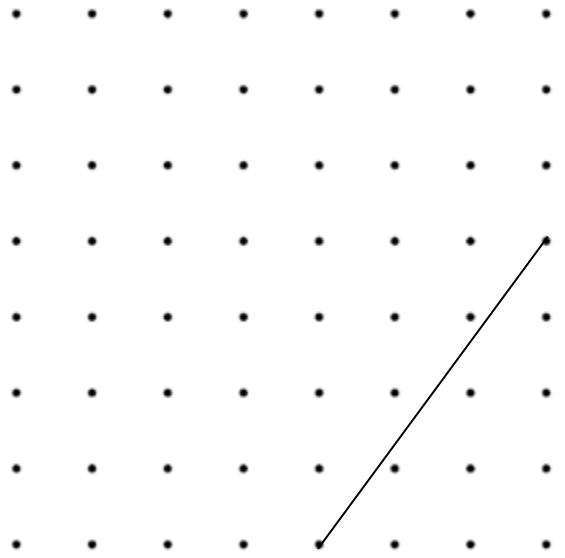
Area of square =_____

Length of the segment = $\sqrt{}$_____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

5)



Area of square =_____

Length of the segment = $\sqrt{}$_____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

Assignment, Part 2

Estimate the following square roots to the nearest tenth. You must show the two perfect squares that the number is between. NO CALCULATOR!!!

1) $\sqrt{84}$ 2) $\sqrt{15}$ 3) $\sqrt{99}$

4) $\sqrt{120}$ 5) $\sqrt{250}$ 6) $\sqrt{44}$

7) $\sqrt{8}$ 8) $\sqrt{80}$ 9) $\sqrt{18}$

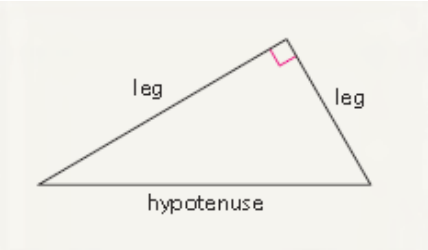
Assignment, Part 3

Identify the sets to which each of the following numbers belongs by marking and "X" in the appropriate boxes.

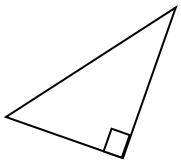
Number	Whole Numbers	Integers	Rational Numbers	Irrational Numbers	Real Numbers
0.3					
8					
$-2.\overline{7}$					
$\sqrt{8}$					
$-\frac{1}{6}$					
$-\sqrt{5}$					

Finding Patterns with the side lengths of Right Triangles

Recall that a right triangle is a triangle with a right, or 90°, angle. The longest side of a right triangle is the side opposite the right angle. We call this side the **hypotenuse** of the triangle. The other two sides are called the **legs.** The right angle of a right triangle is often marked with a square.



Label the legs and the hypotenuse of this right triangle.



Each leg of the right triangle on the left below has a length of 1 unit. Suppose you draw squares on the hypotenuse and legs of the triangle, as shown on the right.

How are the areas of the three squares related?

In this problem, you will look for a relationship among the areas of squares drawn on the sides of right triangles.

Complete the right triangle with the given leg lengths on dot paper. Draw a square on each side of the triangle. Find the areas of the squares and record these results in the table.

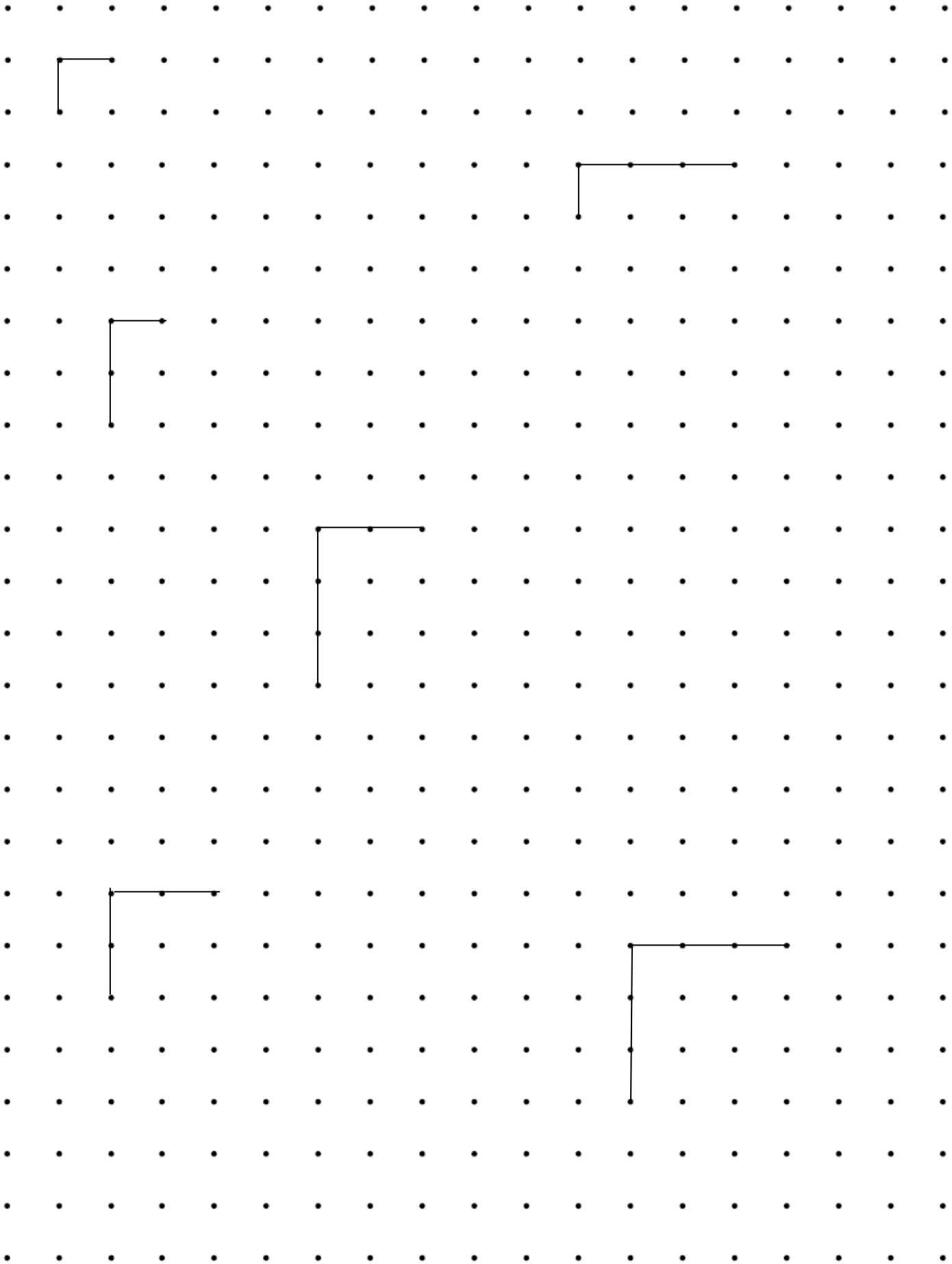
Length of Leg 1	Length of Leg 2	Area of Square on Leg 1	Area of Square on Leg 2	Area of Square on Hypotenuse	Length of Hypotenuse as a Sq. Rt.	Approximate length of Hypotenuse
1	1					
1	2					
2	2					
1	3					
2	3					
3	3					
3	4					
____	____					

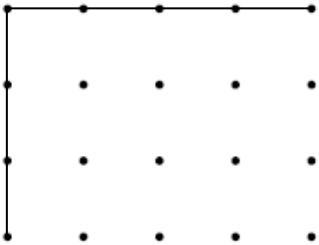
Recall that a conjecture is your best guess about a mathematical relationship. It is usually a generalization about a pattern you think might be true, but you do not yet know for sure to be true.

For each triangle, look for a relationship among the areas of the three squares. Make a conjecture about the areas of squares drawn on any right triangle.

_____

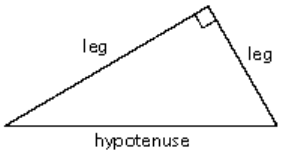
Draw a right triangle with side lengths to test your conjecture. Record these results in the table.





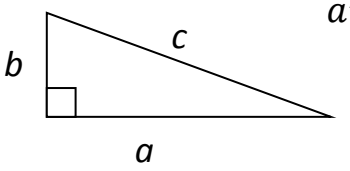
Use the space below to draw the diagram to test your conjecture:

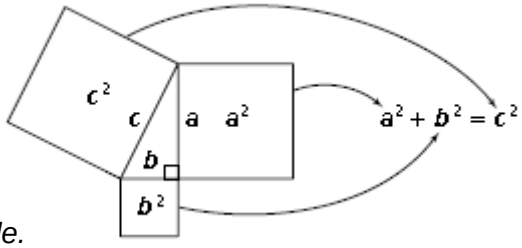
Using the Pythagorean Theorem to Find the Missing Length



In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

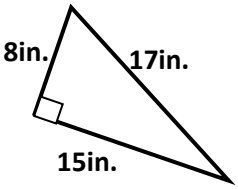
$$a^2 + b^2 = c^2$$

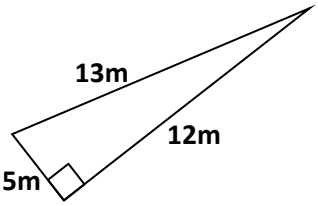


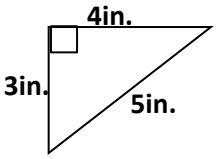


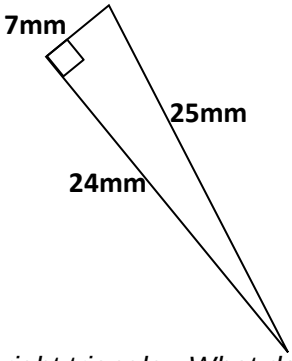
We will use this theorem to find the missing side length of a right triangle.

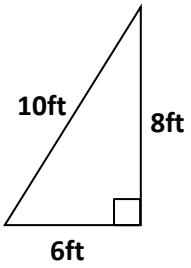
Trace over the legs of the right triangle and circle the measurement of the hypotenuse.





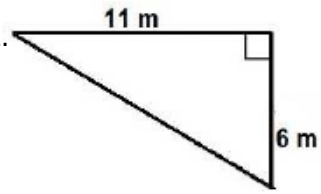






Note: *The _____ is ALWAYS the longest side of the right triangle. What do you think you will need to do if you are missing the leg length instead of the hypotenuse? In the following problems, you will have to decide if you are finding the length of the hypotenuse or a leg length.*

State all lengths as square roots, then approximate to the nearest tenth. If a diagram is not provided, you must draw one including labels.

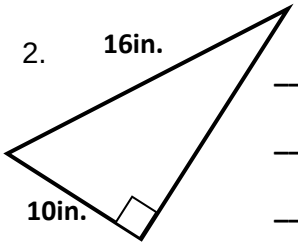
1. 

_____

_____

_____

_____

2. 

_____

_____

_____

_____

3) A computer screen may be described in terms of the diagonal measure of its screen. If a computer screen is 18 inches wide and 11 inches high, what is the length of its diagonal?

[Draw diagram here.]

_____

_____

_____

_____

4) A boat starts at dock and travels 100 km east and then 70 km south. How far is the boat from the dock?

[Draw diagram here.]

5) A 50-ft. cable is stretched from the top of an antenna to an anchor point on the ground 15 ft. from the base of the antenna. How tall is the antenna?

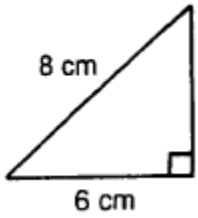
[Draw diagram here.]

Assignment:

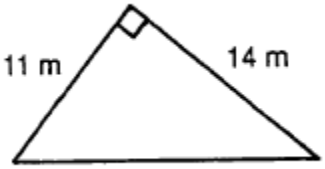
State all lengths as square roots, then approximate to the nearest tenth. If a diagram is not provided, you must draw one including labels.

1

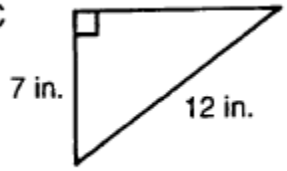
A



B



C



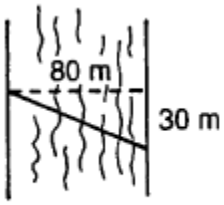
2 Will just bought a big-screen TV. The screen has a diagonal measure of 40 in. If the screen is 32 in. wide, how high is it?

[Draw diagram here.]

3 Jessie leaned a 25-foot ladder against a wall. If the base of the ladder is 7 ft from the wall, how high up the wall will the ladder reach?

[Draw diagram here.]

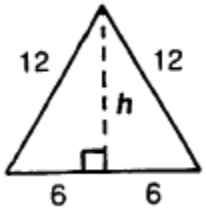
(4) As Greg swam across an 80-meter river, the current carried him 30 m downstream. How far did he swim?



(5) The mast of a sailing ship is 20 ft tall. A rope is stretched 26 ft from the top of the mast to a cleat on the deck of the ship. How far is the cleat from the base of the mast?

[Draw diagram here.]

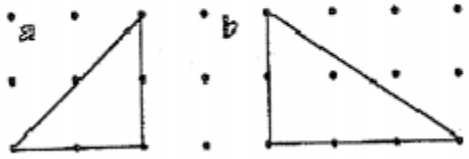
(6) Each side of an equilateral triangle measures 12 cm. Find the height, *h*, of the triangle.



(7) Two jets left an airport at the same time. One traveled east at 300 miles per hour. The other traveled south at 400 miles per hour. How far apart were the jets at the end of an hour?

[Draw diagram here.]

(8) The triangles below are drawn on 1-cm dot paper. Find the perimeter of each triangle.



Hint: You can count the # of cm for the vertical and horizontal distances and then use the Pythagorean Theorem to find the hypotenuse. Don't forget to add the three distances to find the perimeter.

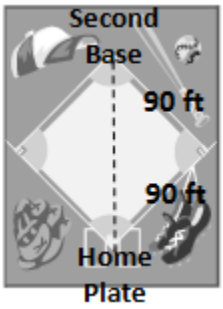
a. _____

Perimeter: _____

b. _____

Perimeter: _____

9) The bases on a baseball diamond are 90 feet apart. How far is it from home plate to second base?



10) A wire is stretched from the top of an 8-ft pole to a bracket 5 ft. from the base of the pole. How long is the wire?

[Draw diagram here.]

11) A park is in the shape of a rectangle 8 miles long and 6 miles wide. How much shorter is your walk if you walk diagonally across the park than along two sides of it?

[Draw diagram here.]

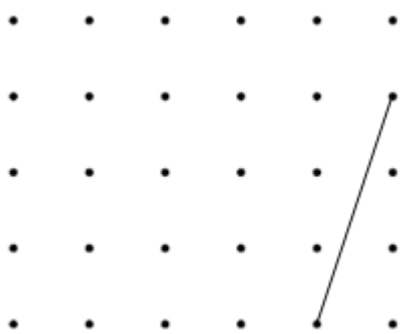
Hint: Go back and answer the original question!

Estimate the following square roots to the nearest tenth. You must show the two perfect squares that the number is between. **NO CALCULATOR!!!**

12) $\sqrt{68}$ 13) $\sqrt{78}$ 14) $\sqrt{51}$ 15) $\sqrt{123}$

16) $\sqrt{287}$ 17) $\sqrt{30}$ 18) $\sqrt{5}$ 19) $\sqrt{47}$

20) Draw a Square. Find the area of the square. Use the area to find the length of the side. Estimate your answer by using a ruler and with a calculator.



Area of square =_____

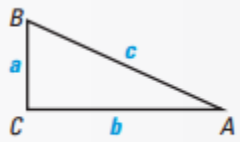
Length of the segment = $\sqrt{\quad}$_____

Length with a ruler ≈_____

Estimate the length with a calculator ≈ _____

The Converse of the Pythagorean Theorem

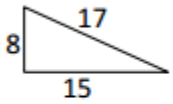
If the square of the length of the longest side of a triangle is equal to the sum of the squares of the lengths of the other two sides, then the triangle is a right triangle.



If $c^2 = a^2 + b^2$, then $\triangle ABC$ is a right triangle.

Is it Right?

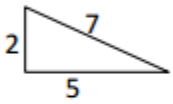
Because of the Pythagorean Converse, we can check whether a triangle is a right triangle or not. Consider the following two triangles. If their side lengths make the Pythagorean Theorem true, they are right.



$8^2 + 15^2 \stackrel{?}{=} 17^2$

$64 + 225 = 289$

True, so this is a right triangle.



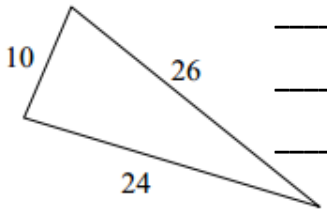
$2^2 + 5^2 \stackrel{?}{=} 7^2$

$4 + 25 \neq 49$

False, $4 + 25$ is not 49, so it is not a right triangle.

Examples

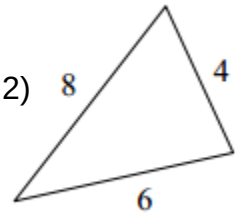
Determine if the following triangles are right triangles or not. You must justify your answer. Diagrams are not drawn to scale.

1) 

_____

_____

_____

2) 

_____

_____

_____

3) a = 5 cm _____

b = 12 cm _____

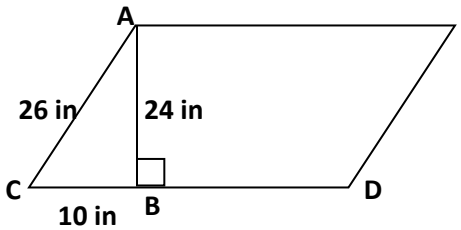
c = 13 cm _____

4) 5 m, 2 m, 3 m _____

_____

_____

5) Determine if $\overline{AB} \perp \overline{CD}$ in the following figure.



_____

_____

_____

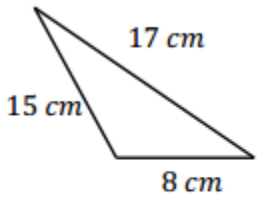
Yes / No

Assignment

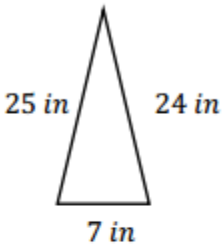
Determine if the following triangles are right triangles or not using the Pythagorean Theorem.

You must justify your answer. Diagrams are not drawn to scale.

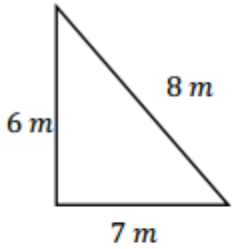
1.



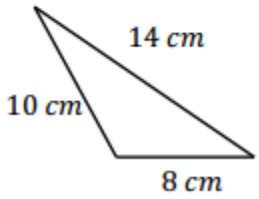
2.



3.



4.



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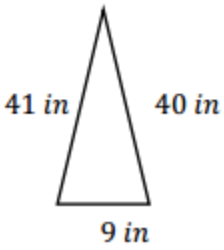

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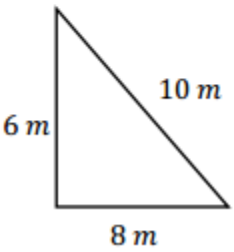


7. $a = 12\ ft$

   $b = 16\ ft$

   $c = 25\ ft$

8. $a = 12\ km$

   $b = 35\ km$

   $c = 37\ km$

9. $a = 10\ mm$

   $b = 24\ mm$

   $c = 27\ mm$

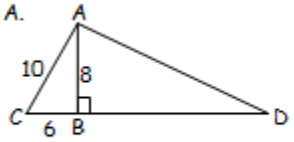
10. 20 ft, 21 ft, 29 ft

11. 17 km, 12 km, 5 km

12. 12 mm, 13 mm, 5 mm

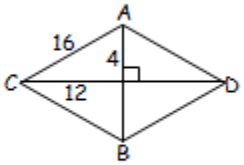
13. Determine which of the following figures $\overline{AB} \perp \overline{CD}$

A.



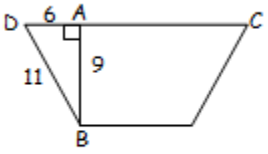
Yes / No

B.



Yes / No

C.

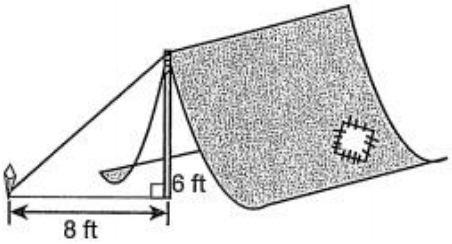


Yes / No

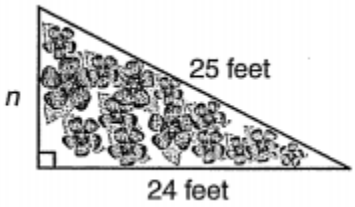
Review Using the Pythagorean Theorem

State all lengths as square roots, then approximate to the nearest hundredth. If a diagram is not provided, you must draw one.

1) A tent is supported by a guy rope tied to a stake, as shown in the diagram. What is the the length of the rope? _____

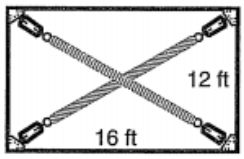


2) Stephanie is planning a right triangular garden. She marked two sides that measure 24 feet and 25 feet. What is the length of side *n*? _____

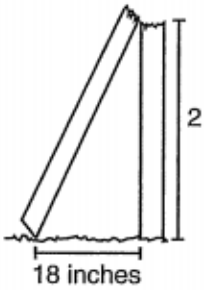


3) A builder needs to add diagonal braces to a wall. The wall is 16 feet wide by 12 feet high. What is the length of each brace?

_____



4) The diagram at the right shows how a post was broken. What was the original height of the post? _____



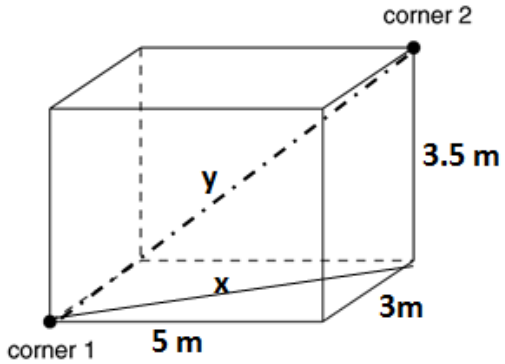
Answer the question asked:

Caution!
Pay attention to the units!!

5) The bases on a softball diamond are 60 feet apart. How far is it from home plate to second base?

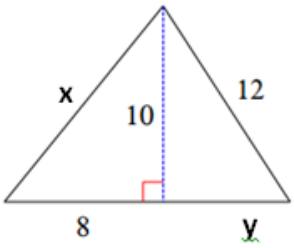
[Draw diagram here.]

6) A room is 5 m by 3 m and has a height of 3.5 m. Find the distance from a corner point on the floor to the opposite corner of the ceiling.



Solution: _____

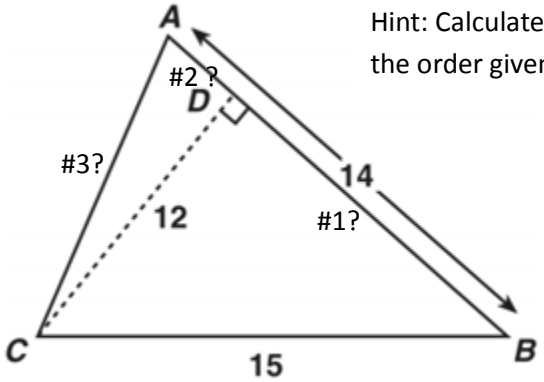
7) Find the area and perimeter of the triangle. Round intermediate values to the nearest tenth. Use the rounded values to calculate the next value. Round your final answer to the nearest tenth. Assume units are in feet.



$A = \frac{1}{2}bh$

P =

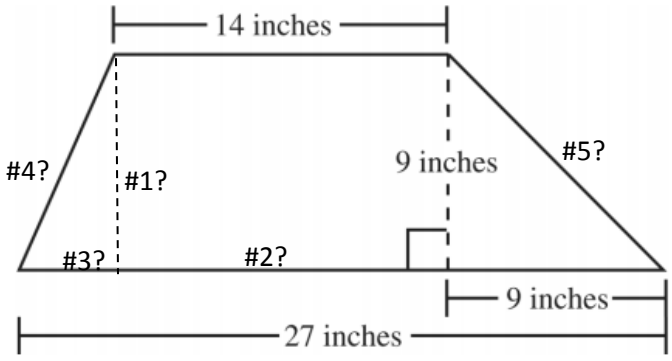
8) In the figure below, $\overline{AB}$ and $\overline{CD}$ are perpendicular.



Hint: Calculate the ?'s in the order given.

What is the perimeter of $\triangle ABC$?

9) The trapezoid pictured below has the measurements shown.



What is the perimeter of the trapezoid?