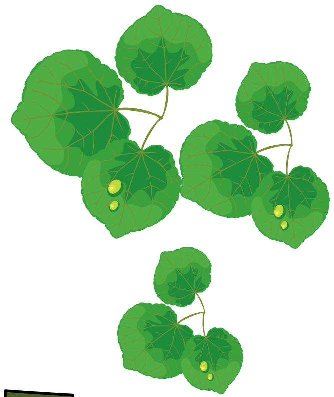
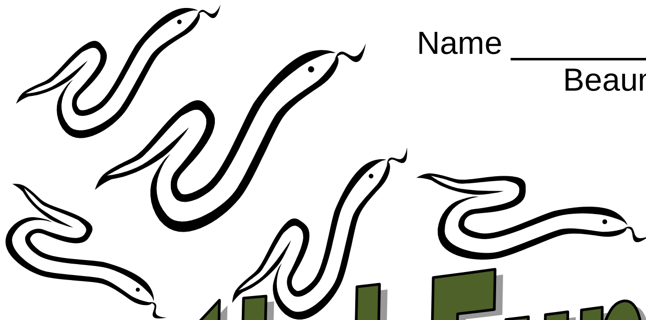


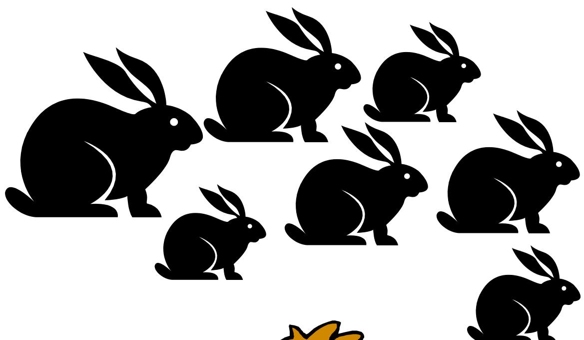


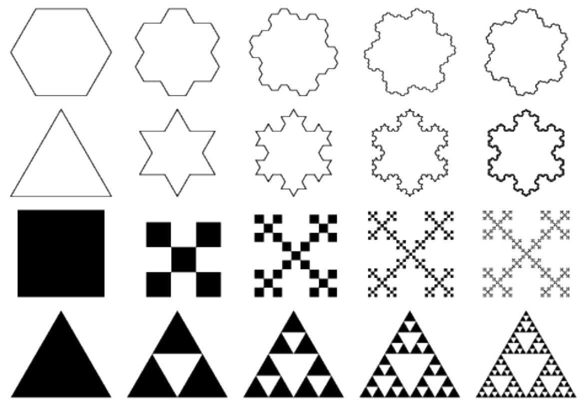
Name ______________________________

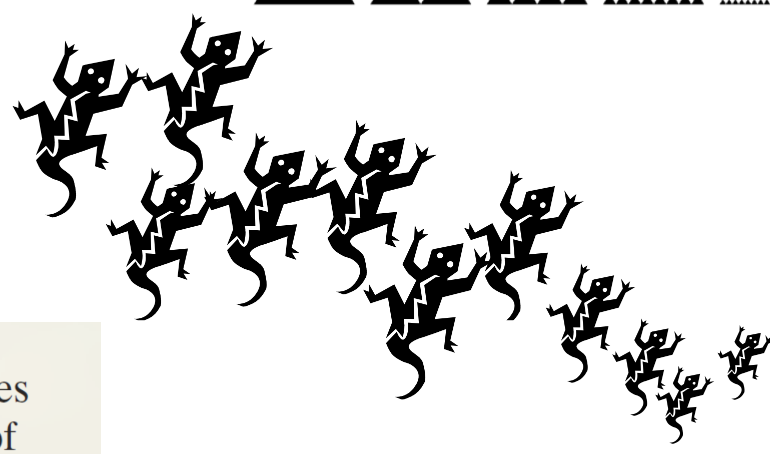
Beaumont Middle School 2017-2018

Adv. Algebra I

Exponential Functions

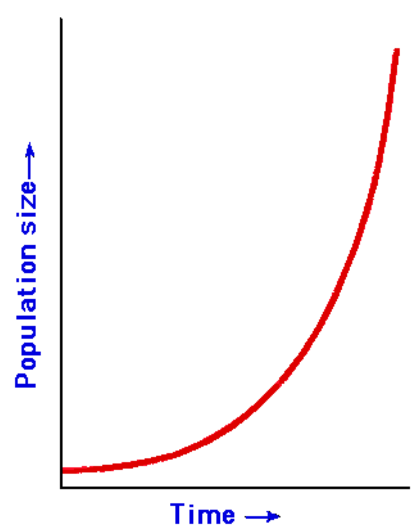




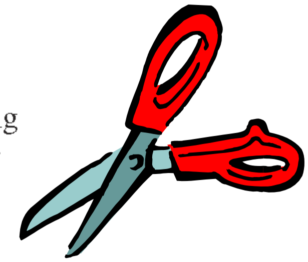


One of the most important uses of algebra is to model patterns of change. You are already familiar with linear patterns of change. Linear patterns have constant differences and straight-line graphs. In a linear relationship, the *y*-value increases by a constant amount each time the *x*-value increases by 1.

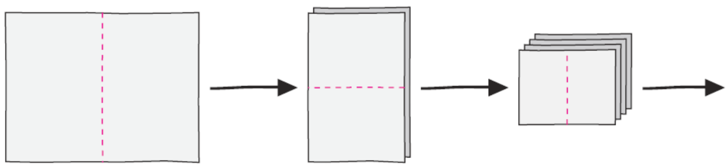
In this unit, you will study exponential patterns of change.



Growing, Growing, Growing: Investigation 1



Chen, the secretary of the Student Government Association, is making ballots for tonight's meeting. He starts by cutting a sheet of paper in half. He then stacks the two pieces and cuts them in half. He stacks the resulting four pieces and cuts them in half. He repeats this process, creating smaller and smaller pieces of paper.



After each cut, Chen counts the ballots and records the results in a table.

Number of Cuts	Number of Ballots
0	1
1	2
2	4
3	
4	

Chen wants to predict the number of ballots after any number of cuts.

1. Predict how many ballots will result from 3 cuts. _________
2. Predict how many ballots will result from 4 cuts. _________
3. Predict how many ballots will result from 10 cuts. _________

4. Complete the 2^{nd} **column** in the table to show the number of ballots after each of the cuts.

Number of Cuts (n)	**Number of Ballots (b)**		**Shortcut Form for Number of Ballots using Exponents (b)**
0	**1**		
1	**2**		
2	**4**		
3	**8**		
4			
5			
6			
7			
8			
9			
10			

5. How did you find your entries in the table?____

6. Fully complete the table above.

7. What is the relationship between the number of cuts and the number of ballots? (In other words, how can you use the number of cuts to figure out the number of ballots?)

____

8. Write a rule (equation) to explain the relationship between of the number of cuts (*n*) and the number of ballots (*b*).

____

9. Use your rule (equation) to determine how many ballots Chen would have if he made 20 cuts?

__________ = __________

10. Use your rule (equation) to determine how many ballots Chen would have if he made 30 cuts?

__________ = __________

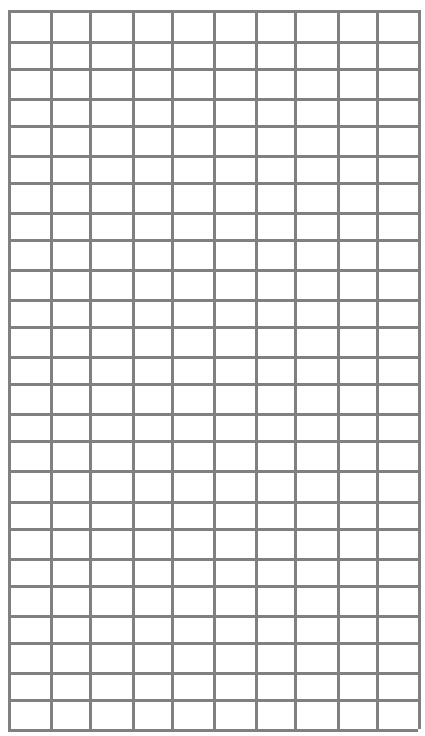
11. How many cuts would it take to make enough ballots for all 500 students in Chen's school? ________

Explain how you determined this answer. ____

____

12. Graph the relationship using an interval of 1 on the x-axis and 50 on the y-axis.

Number of Cuts	Number of Ballots
0	1
1	2
2	4
3	
4	



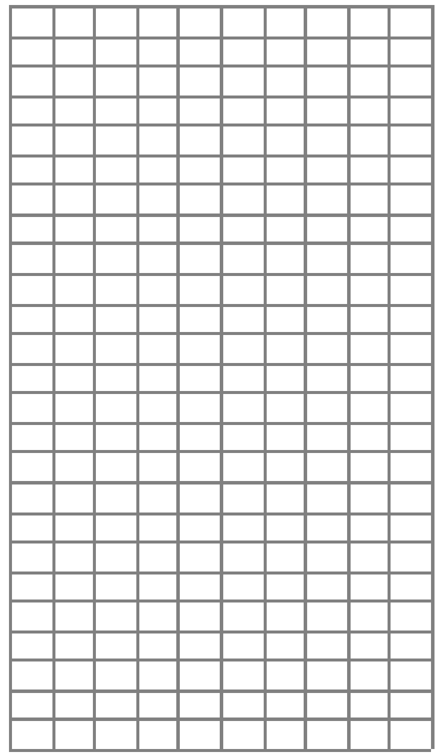
Bacteria Growth 

If you don't brush your teeth regularly, it won't take long for large colonies of bacteria to grow in your mouth. Suppose that a single bacterium lands on one of your teeth and starts reproducing by a factor of four every hour. (multiplies by four every hour)

b = the number of bacteria Equation: **$b = 1 * 4^t$**

t = the number of hours

1. Complete the table and graph for the relationship.
Use an interval of 1 on the x-axis and 50 on the y-axis.



2. Use the equation to find how many bacteria will be in the new colony after 12 hours?

____

3. How many bacteria will be in the new colony after 13 hours? __________
Explain how you can use your answer from #2 instead of using your equation.

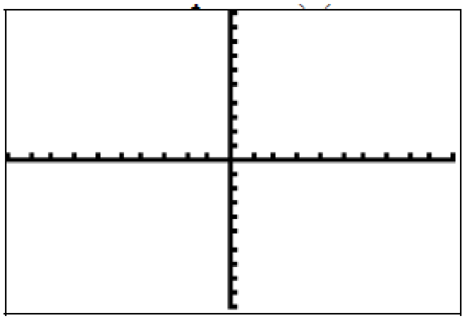
____

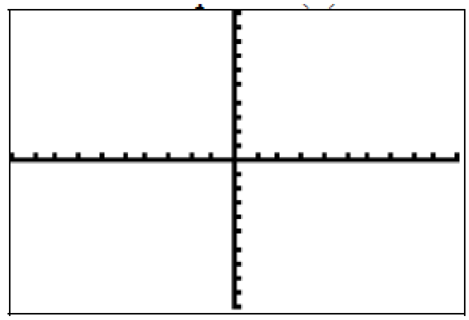
4. After how many hours will there be at least 1 million bacteria in the colony ? ___________
(Use guess and check to find your answer.)

Using a graphing calculator to complete the table and graph each of the following functions in a standard window.

1) $y = 2^x$

x	y
0	
1	
2	
3	



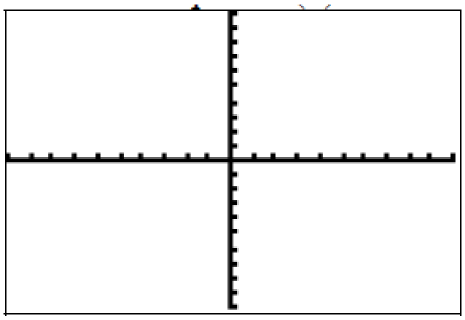


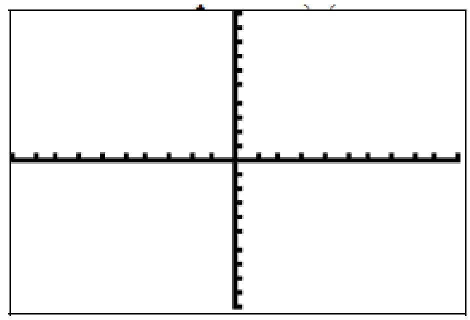
2) $y = 3^x$

x	y
0	
1	
2	
3	

3) $y = (2^x)4$

x	y
0	
1	
2	
3	



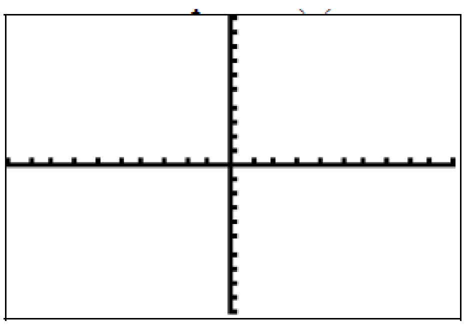


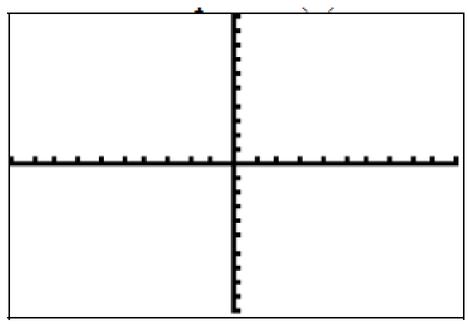
4) $y = (3^x)4$

x	y
0	
1	
2	
3	

5) $y = (2^x)6$

x	y
0	
1	
2	
3	



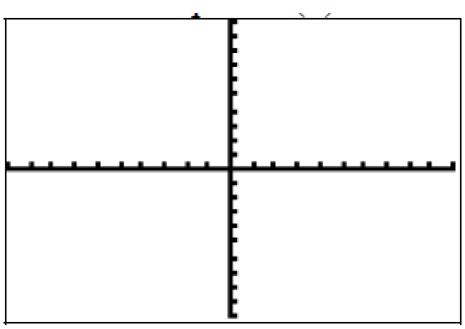


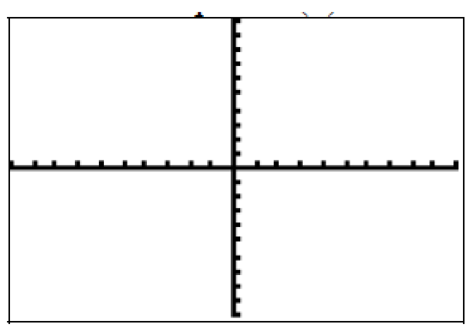
6) $y = (3^x)6$

x	y
0	
1	
2	
3	

7) $y = (4^x)3$

x	y
0	
1	
2	
3	





8) $y = (4^x)5$

x	y
0	
1	
2	
3	

a) All of these functions are ______________________ in shape.

b) If an equation is in the form $y = (a^x)b$, then a affects the ____________________ of the graph.

c) If an equation is in the form $y = (a^x)b$, then b is the ____________________ of the graph.

Exponential Equations

$y = a^x * b$

Growth Factor (Pattern: # you multiply as x values increase)

y-intercept (where x = 0 or starting #)

Growing, Growing, Growing: Investigation 2

1. Ghost Lake is a popular site for fisherman, campers, and boaters. In recent years, a certain water plant has been growing on the lake at an alarming rate. The surface area of Ghost Lake is 25,000,000 square feet. At present, 1,000 square feet are covered by the plant. The Dept. of Natural Resources estimates that the area is doubling every month.

A) 1. Explain what information the variables and numbers in the situation represent.

Numbers:____

Variables: ____

2. Write an equation that represents the growth pattern of the plant on Ghost Lake.
(Let *m* = # of months and *a* = the plant area) _______________

3. How does this compare to the equation for yesterday's bacteria growth problem?

____

____

B) Make a table and a graph of the equation. (Use x-intervals of 1 and y-intervals of 1,000.)

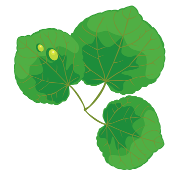
C) How much of the lake's surface will be covered with the water plant by the end of a year? (Show your work.)

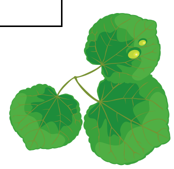
____

D) In how many months will the plant completely cover the surface of the lake? _____________

2. Loon Lake has a “killer plant” problem similar to Ghost Lake. Currently, 5,000 square feet of the lake is covered with the plant. The area covered is growing by a factor of 1.5 each year.

a) Complete the table and graph to show the area covered by the plant for the next 5 years. (Use intervals of 1 for the x-axis and 1,000 for the y-axis.)





b) Write an equation that represents the growth pattern of the plant on Loon Lake.
(Let *t* = # of years and *a* = the plant area) ____________________

c) The surface area of the lake is approximately 200,000 square feet. How long will it take before the lake is completely covered?

____

3. In parts of the U.S., wolves are being reintroduced to wilderness areas where they have become extinct. Suppose 20 wolves were released in Northern Michigan, and the yearly growth factor for this population is expected to be 1.2.

a) Make a table showing the projected number of wolves at the end of each of the first 6 years.

b) Write an equation that models the growth pattern of the wolf population. ________________

c) How long will it take for the new population to exceed 100? ______________________________

4. Fruit flies are often used in genetic experiments because they reproduce at a phenomenal rate. In 12 days, a pair of *Drosophila* can mature and produce a new generation of fruit flies. The table below shows the number of fruit flies in three generations of a laboratory colony.

Generation	0	1	2	3
# of Flies	2	120	7200	432,000

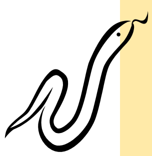
a) What is the growth factor for this fruit-fly population? ____________

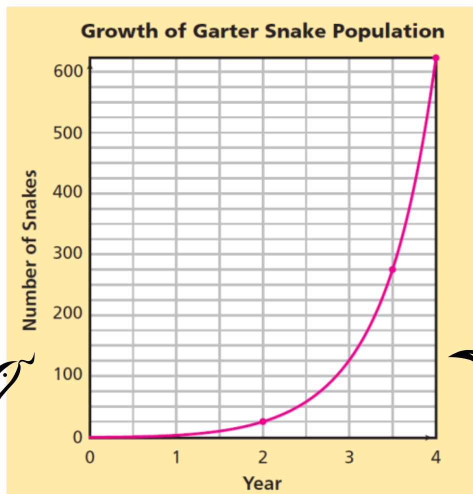
Show how you found the answer. ____

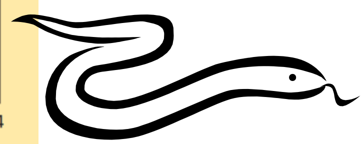
b) Write an equation that models this situation. _______________

If this pattern continues, how many fruit flies will there be in the fifth generation? ________________

5) The graph shows the growth of a garter snake population after it was introduced to a new area. The population is growing exponentially.







A) 1. Find the snake population for year 2 ____________________________

year 3 ____________________________

year 4 ____________________________

2. Use the pattern in your answer from part 1 to estimate the population in year 1 _____________

3. Find the growth factor for the population. ________________________________

4. Show how you can find the y-intercept for the graph. ______________________________

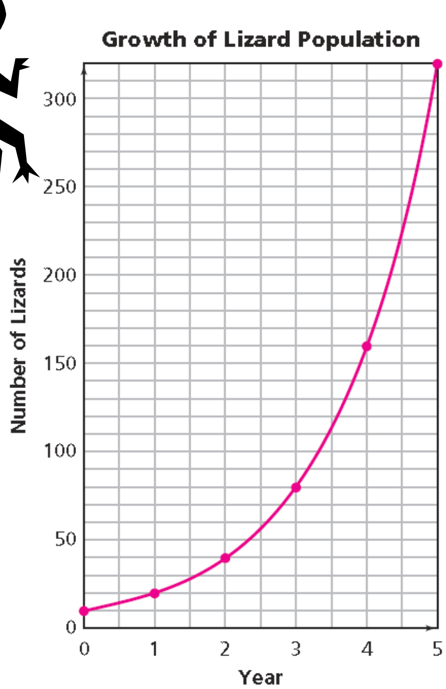
C) Write an equation relating time *t* in years and population *p*. Explain what information the numbers in the equation represent.

____

____

D) In what year is the population likely to reach 1,500? __________________________

6. The following graph represents the population growth of a certain kind of lizard.



a) What information does the point (2,40) on the graph tell you?

____

b) What information does the point (1,20) on the graph tell you?

____

c) What is the growth factor of the population? (Show your work.)

____

d) What was the initial population? ______________

e) Write an equation relating time *t* in years and population.

____

f) When will the population exceed 100 lizards? __________

g) When will the population exceed 600 lizards? __________

Linear and Exponential Comparison

<u>Linear Equations</u>

$y = mx + b$

Rate or slope (Pattern: # you <u>add</u> as x values increase)

y-intercept (where x = 0 or starting #)

<u>Exponential Equations</u>

$y = a^x * b$

Growth Factor (Pattern: # you <u>multiply</u> as x values increase)

y-intercept (where x = 0 or starting #)

7)

x	0	1	2	3
y	2	6	18	54

Growth factor: _______ Y-intercept: ______ Equation: __________________

8)

x	0	1	2	3
y	4	40	400	4000

Growth factor: _______ Y-intercept: ______ Equation: __________________

9)

x	0	1	2	3
y	7	28	112	448

Growth factor: _______ Y-intercept: ______ Equation: __________________

10)

x	0	1	2	3
y		12	96	768

Growth factor: _______ Y-intercept: ______ Equation: __________________

Growing, Growing, Growing Investigation 3.1 and Scientific Notation

3.1 Reproducing Rabbits

1. In 1859, a small number of rabbits were introduced to Australia by English settlers. The rabbits had no natural predators in Austrailia, so they reproduced rapidly and became a serious problem, eating grasses intended for sheep and cattle.

If biologists had counted the rabbits in Australia in the years after they were introduced, they might have collected data like these:

A. The table shows the rabbit population growing exponentially.

Growth of Rabbit Population

Time (yr)	Population
0	100
1	180
2	325
3	583
4	1,050

1. What is the growth factor? _______ Show how you found your answer.______________

2. Assume the growth pattern continued. Write an equation for the rabbit population, *p*, for any year, *n*, after the rabbits were first introduced. Explain what the numbers in your equation represent.

____

3. How many rabbits will there be in 10 years? ________________

How many will there be after 25 years? ______________ After 50 years?_________________

4. After how many years will the rabbit population exceed one million? ____________________

++

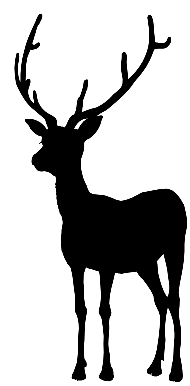
B. Suppose that during a different time period, the rabbit population could be predicted by the equation $p = 15\ (1.2^n)$, where p is the population in millions, and n is the number of years.

1. What is the growth factor? ______________ 2. What is the initial population? ___________

2. The table shows that the elk population in a state forest is growing exponentially.

Growth of Elk Population

Time (yr)	Population
0	30
1	57
2	108
3	206
4	391
5	743



a. What is the growth factor? ____________

Show how you got your answer. __________________

b. Suppose that this growth patterns continues.

How many elk will there be in 10 years? _______15 years? ____

c. Write an equation you could use to predict the elk population p for any year n after the elk were first counted.

____

d. After how many years will the elk population exceed one million? ____________________

3. As a biology experiment, Amanda is studying the growth of a beetle population. She starts her experiment with 5 beetles. The next month she counts 15 beetles.

a. Suppose the population is growing **linearly**. What is the <u>pattern</u> in the population from one month to the next? _______________

b. Suppose the population is growing **linearly**. Complete the table.

# of Months	# of Beetles
0	
1	
2	
3	
4	

c. Write an equation for the number of beetles, *b*, after *m* months if the beetle population is growing **linearly**. _________________________

d. How long will it take the population to reach 200 if it is growing **linearly**? ________________

e. Suppose the population is growing **exponentially**. What is the <u>pattern</u> in the population from one month to the next? _______________

f. Suppose the population is growing **exponentially**. Complete the table.

# of Months	# of Beetles
0	
1	
2	
3	
4	

g. Write an equation for the number of beetles, *b*, after *m* months if the beetle population is growing **exponentially**. ________________________

h. How long will it take the population to reach 200 if it is growing **exponentially**? _______________

4. Each table shows exponential growth or a linear relationship. Complete each table. Fill in the blanks.

Table A

t	0	1	2	3	4
m	8	24	72	216	

Linear or exponential?________________ Equation:_____________________________

Table B

t	0	1	2	3	4
p	15	18	21	24	

Linear or exponential?________________ Equation:_____________________________

Table C

n	0	1	2	3	4	10
a	5	7.5	11.25	16.88		

Linear or exponential?________________ Equation:_____________________________

Table D

s	0	1	2	3	4	20
m		8.5	6	3.5		

Linear or exponential?________________ Equation:_____________________________

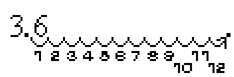
Scientific Notation and Standard Form (Decimal Notation) Notes

- By using exponents, we can reformat numbers. For very large or very small numbers, it is sometimes simpler to use "scientific notation" (so called, because scientists often deal with very large and very small numbers).
- The format for writing a number in scientific notation is fairly simple: (first digit of the number) followed by (the decimal point) and then (all the rest of the digits of the number), times (10 to an appropriate power). The conversion is fairly simple.

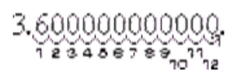
- **Write 124 in scientific notation.**
  This is not a very large number, but it will work nicely for an example. To convert this to scientific notation, I first write "1.24". This is not the same number, but (1.24)(100) = 124 is, and $100 = 10^2$. Then, in scientific notation, 124 is written as **1.24×10^2.**

- Actually, converting between "regular" notation and scientific notation is even simpler than I just showed; because all you really need to do is count decimal places.

- **Write in decimal notation: 3.6×10^{12}**
  Since the exponent on 10 is positive, I know they are looking for a LARGE number, so I'll need to move the decimal point to the right, in order to make the number LARGER. Since the exponent on 10 is "12", I'll need to move the decimal point twelve places over. First, I'll move the decimal point twelve places over. I make little loops when I count off the places, to keep track:



Then I fill in the loops with zeroes:



In other words, the number is **3,600,000,000,000**, or 3.6 trillion

- **Convert 93,000,000 to scientific notation.**
  This is a large number, so the exponent on 10 will be positive. The first "interesting" digit in this number is the leading 9, so that's where the decimal point will need to go. To get from where it is to right after the 9, the decimal point will need to move seven places to the left. Then the power on 10 will be a positive 7, and the answer is **9.3×10^7**

Scientific Notation and Standard Form (Decimal Notation) Practice

Write in standard form.

1) 4.0×10^3 ________________ 2) 4.5×10^4 ________________ 3) 6.5×10^5 ________________

4) 7.6×10^2 ________________ 5) 8×10^3 ________________ 6) 6.32×10^7 ________________

Write each number in scientific notation.

7) 465,000,000________________ 8) 98,000,000,000________________ 9) 373,000 ________________

10) 697,000,000,000______________ 11) 54,000,000 ________________ 12) 24,340,000________________

Use your calculator to evaluate the following. Write the answer in scientific notation and standard form. Round to three significant digits.

	Scientific notation	Standard form
13) 7^{12}	____________________	____________________
14) 12^{15}	____________________	____________________
15) 4^{24}	____________________	____________________
16) 18^{9}	____________________	____________________

Growing, Growing, Growing: Practice Inv. 1 - 3.1

1. **M**old can spread rapidly. For example, the area covered by mold on a loaf of bread left out in warm weather grows exponentially.

Students at Magnolia Middle School conducted an experiment. They set out a shallow pan containing a mixture of chicken bouillon (BOOL yahn), gelatin, and water. Each day, the students recorded the area of the mold in square millimeters.

The students wrote the exponential equation $m = 50(3^d)$ to model the growth of the mold. In this equation, m is the area of the mold in square millimeters after d days.

A. What is the area of the mold at the start of the experiment? ______________

B. What is the growth factor? ______________

C. What is the area of the mold after 5 days? ______________

D. On which day will the area of the mold reach 6,400 mm^2? ______________

E. An exponential equation can be written in the form $y = a(b^x)$, where a and b are constant values.

1. What value does b have in the mold equation? What does this value represent? ______________________________

2. What value does a have in the mold equation? What does this value represent? ______________________________

2. A population of mice has a growth factor of 3. After 1 month, there are 36 mice. After 2 months, there are 108 mice.

a. How many mice were in the population initially (at 0 months)? ______________

b. Write an equation for the population after any number of months. Explain what information the numbers and variables in your equation represent. ______________________________

c. Make a table showing the population of mice for the first 5 months.

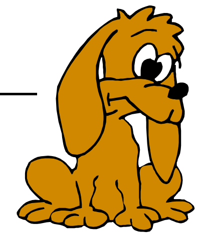
d. Graph the population growth for 4 months. (Use interval of 1 on the x axis, 100 on the y axis.)

3. Fido did not have fleas when his owners took him to the kennel. The number of fleas on Fido after he returned from the kennel grew according to the equation $f = 8(3^n)$, where f is the number of fleas and n is the number of weeks since he returned from the kennel. (Fido left the kennel at week 0.)

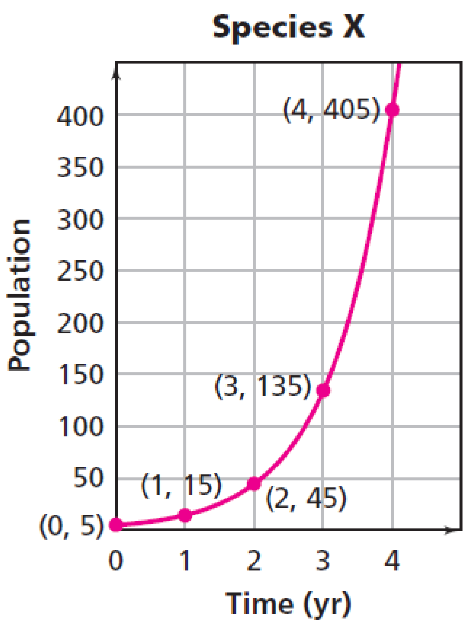
a. How many fleas did Fido pick up at the kennel? ______________

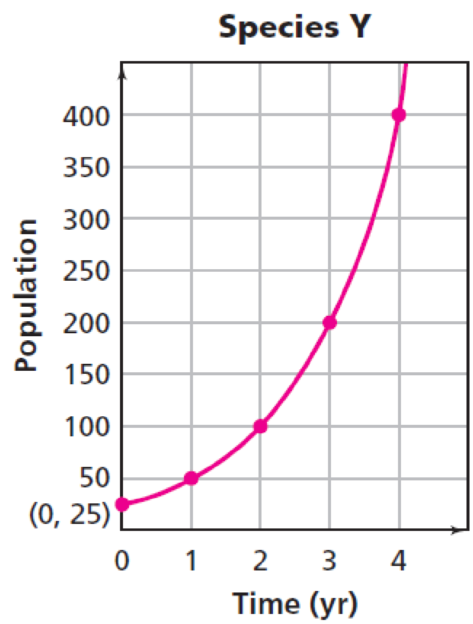
b. What is the growth factor for the number of fleas? ______________

c. How many fleas will Fido have after 10 weeks if he is not treated? ___________________



4.





a. Find the growth factors for the two species. Which species is growing faster? Explain. ________________________

b. What are the y-intercepts for the graphs of Species X and Species Y? Explain what these y-intercepts tell you about the populations. ________________

c. Write an equation that describes the growth of Species X. ________________

d. Write an equation that describes the growth of Species Y. ________________

e. For which equation is (5, 1215) a solution? ________________

5. **Multiple Choice** Choose the answer that best approximates 3^{20} in scientific notation.

A. 3.5×10^{-9} **B.** 8×10^3 **C.** 3×10^9 **D.** 3.5×10^9

6. **Multiple Choice** Choose the answer that is closest to 2.575×10^6.

F. 2^{18} **G.** 12^6 **H.** 6^{12} **J.** 11^9

Indicate whether each table is linear or exponential. Complete each table. Write an equation for each relationship.

1) Linear or Exponential? Equation: ______________________________

x	0	1	2	3	4
y	10	25	62.5	156.25	

2) Linear or Exponential? Equation: ______________________________

x	0	1	2	3	4
y	10	25	40	55	

3) Linear or Exponential? Equation: ______________________________

x	0	1	2	3	4	10
y	4	4.8	5.76	6.912		

4) Linear or Exponential? Equation: ______________________________

x	0	1	2	3	4	20
y	50	60	70	80		

5) Linear or Exponential? Equation: ______________________________

x	0	1	2	3	4	20
y		10	20	40		

Scientific Notation and Standard Form (Decimal Notation) Practice

Write in standard form.

1) 6.0×10^7 ______________ **2) 4.52×10^6** ______________ **3) 3.121×10^5** ______________

4) 8.5×10^6 ______________ **5) 5×10^7** ______________ **6) 3.256×10** ______________

Write each number in scientific notation.

7) 825,000,000______________ **8) 42,000,000,000**______________ **9) 846,000,000**______________

10) 156,000,000______________ **11) 49,000,000,000**______________ **12) 29,450,000**______________

Use your calculator to evaluate the following. Write the answer in scientific notation and standard form. Round to three significant digits.

	Scientific notation	**Standard form**
13) 8^{15}	______________	______________
14) 14^{16}	______________	______________
15) 6^{20}	______________	______________
16) 11^{12}	______________	______________

Growing, Growing, Growing Investigation 3.2 and More Scientific Notation

3.2 Investing for the Future

The yearly growth factor for our rabbit population from Investigation 3.1 was 1.8. Suppose the population data fit the equation $P = 100(1.8)^n$ exactly. Then its table would look like the one below.

Growth of Rabbit Population

Time (yr)	Population
0	100
1	180
2	325
3	583
4	1,050

The growth factor of 1.8 is the number by which the population for year *n* is multiplied to get the population for the next year, *n* + 1.
You can think of the growth factor in terms of a percent change. To find the percent change, compare the difference in population for two consecutive years, *n* and *n* + 1, with the population of year *n*.

- From year 0 to year 1, the percent change is $\frac{180-100}{100} = \frac{80}{100} = 80\%$.

  The population of 100 rabbits in year 0 *increased* by 80%, resulting in 100 rabbits (80%) = 80 additional rabbits.
- From year 1 to year to the percent change is $\frac{324-180}{100} = \frac{144}{180} = 80\%$.

  The population of 180 rabbits in year 1 *increased* by 80%, resulting in 180 rabbits(80%) = 144 additional rabbits.

The percent increase is called the ***growth rate***. In some growth situations, the growth rate is given instead of the growth factor. For example, changes in the value of investments are often expressed as percents.

PROBLEM 3.2

When Sam was in 7th grade, his aunt gave him a stamp collection worth $2500. Sam considered selling the collection, but his aunt told him that, if he saved it, it would increase in value.

A. Sam saved the collection, and its value increased by 6% each year for several years in a row.

1. Make a table showing the value of the collection each year for the five years after Sam's aunt gave it to him. (round to nearest dollar.)

Sam's Stamp Collection at 6%

Year	Value
0	$2,500
1	
2	
3	
4	
5	

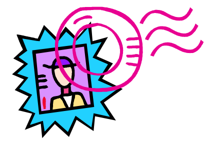
2. Look at the pattern of growth from one year to the next. Is the value growing exponentially? ____________

3. Write an equation for the value *v* of the collection after *n* years. ______________________

B. Suppose the value of the stamps increased by 4% each year instead of by 6%.

1. Make a table showing the value of the collection each year for the five years after Sam's aunt gave it to him. (round to nearest dollar.)

Sam's Stamp Collection at 4%



Year	Value
0	$2,500
1	
2	
3	
4	
5	

2. What is the growth factor from one year to the next? ___________________

3. Write an equation for the value *v* of the collection after *n* years. ______________________

C. 1. Find the growth factor associated with each growth rate.

a) 5% ____________ b) 15% _____________ c) 30% ______________

d) 75% ___________ e) 100% ____________ f) 150% _____________

2. How can you find the growth factor if you know the growth rate? _________________________

____

D. 1. Find the growth rate associated with each growth factor.

a) 1.5 _____________ b) 1.06 ______________ c) 1.1 _______________

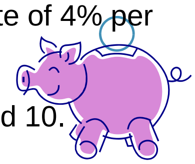
d) 1.05 _____________ e) 1.25 ______________ f) 1.8 _______________

2. How can you find the growth rate if you know the growth factor? _______________________

____

<u>Homework:</u>

1. Maya's grandfather opened a saving account for her when she was born. He opened the account with $100 and did not add or take out any money after that. The money in the account grows at a rate of 4% per year.



a. Make a table to show the amount in the account from the time Maya was born until she turned 10. (Round to the nearest dollar.)

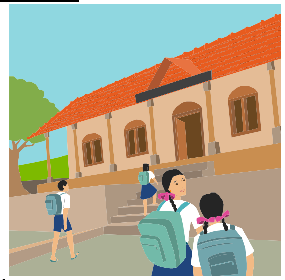
AGE											
VALUE											

b. What is the growth factor for the account? _____________

c. Write an equation for the value of the account after any number of years. ___________________

2. Currently, 1,000 students attend Greenville Middle School. The school can accommodate 1,300 students. The school board estimates that the student population will grow by 5% per year for the next several years.

a. In how many years will the population outgrow the present building? ___________________



b. Suppose the school limits its growth to 50 students per year.

Write an equation to model this situation. ____________________

How many years will it take for the population to outgrow the school? (Show your work.)

____

3. Suppose that, for several years, the number of radios sold in the U.S. increased by 3% per year.

a. Supposed one million radios sold in the first year of this time period. Make a table showing about how many radios would be sold in each of the next 6 years.

b. Suppose only 100,000 radios sold in the first year. About how many radios sold in each of the next six years?

Year	Radios Sold

Year	Radios Sold

4. Find the growth rate associated with the given growth factor.

a. 1.4 ______________ b. 1.09 ______________ c. 1.75 _______________

d. 1.02 ______________ e. 1.85 ______________ f. 1.7 _______________

5. Find the growth factor associated with the given rate.

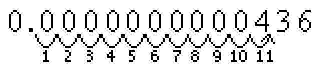
a. 45% ______________ b. 90% ______________ c. 3% _______________

d. 20% ______________ e. 15% ______________ f. 4% _______________

Scientific Notation and Standard Form (Decimal Notation) Notes

- By using exponents, we can reformat numbers. For very large or very **small** numbers, it is sometimes simpler to use "scientific notation" (so called, because scientists often deal with very large and very small numbers).
- The format for writing a number in scientific notation is fairly simple: (first digit of the number) followed by (the decimal point) and then (all the rest of the digits of the number), times (10 to an appropriate power). The conversion is fairly simple.

- **Write 0.000 000 000 043 6 in scientific notation.**
  In scientific notation, the number part (as opposed to the ten-to-a-power part) will be "4.36". So I will count how many places the decimal point has to move to get from where it is now to where it needs to be:

  Then the power on 10 has to be –11: "eleven", because that's how many places the decimal point needs to be moved, and "negative", because I'm dealing with a SMALL number. So, in scientific notation, the number is written as **4.36×10^{-11}**

- **Convert 4.2×10^{-7} to decimal notation.**
  Since the exponent on 10 is negative, I am looking for a small number. Since the exponent is a seven, I will be moving the decimal point seven places. Since I need to move the point to get a *small* number, I'll be moving it to the left. The answer is **0.000 000 42**

- **Convert 0.000 000 005 78 to scientific notation.**
  This is a small number, so the exponent on 10 will be negative. The first "interesting" digit in this number is the 5, so that's where the decimal point will need to go. To get from where it is to right after the 5, the decimal point will need to move nine places to the right. Then the power on 10 will be a negative 9, and the answer is **5.78×10^{-9}**

Just remember: However many spaces you moved the decimal, that's the power on 10. If you have a small number (smaller than 1, in absolute value), then the power is negative; if it's a large number (bigger than 1, in absolute value), then the exponent is positive.

Warning: A negative on an exponent and a negative on a number mean two very different things! For instance:

$-0.00036 = -3.6 \times 10^{-4}$
$0.00036 = 3.6 \times 10^{-4}$
$36{,}000 = 3.6 \times 10^{4}$
$-36{,}000 = -3.6 \times 10^{4}$

Don't confuse these!

Write in standard form.

1) 4.82×10^{-5} ________________ 2) 2.6×10^{-7} ________________ 3) 1.79×10^{-4} ________________

4) 5.28×10^{5} ________________ 5) 7×10^{8} ________________ 6) 6.12×10^{6} ________________

Write each number in scientific notation.

7) 0.000 000 000 52 ____________ 8) 0.000 000 041 ______________ 9) 0.000 000 398 ______________

10) 578,000,000______________ 11) 38,000,000,000 ___________ 12) 219,000 __________________

Use your calculator to evaluate the following. Write the answer in scientific notation and standard form. **Round to three significant digits.**

	Scientific notation	Standard form
13) 5^{-11}	______________	______________
14) 9^{-12}	______________	______________
15) 8^{-15}	______________	______________
16) 5^{20}	______________	______________
17) 9^{11}	______________	______________
18) 10^{12}	______________	______________

Indicate whether each table is linear or exponential. Complete each table. Write an equation for each relationship.

1) Linear or Exponential? Equation: ______________________

x	0	1	2	3	4
y	6	7.5	9	10.5	

2) Linear or Exponential? Equation: ______________________

x	0	1	2	3	4
y	8	12	18	27	

3) Linear or Exponential? Equation: ______________________

x	0	1	2	3	4	**10**
y	9	17.1	32.49	61.731		

4) Linear or Exponential? Equation: ______________________

x	0	1	2	3	4	**20**
y	4	6.8	9.6	12.4		

5) Linear or Exponential? Equation: ______________________

x	0	1	2	3	4	**20**
y		7	24.5	85.75		

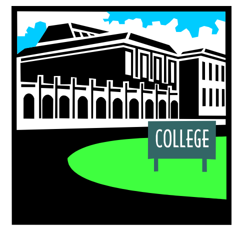
Growing, Growing, Growing Investigation 3.3

3.3 Making a Difference

In Problem 3.2, the value of Sam's stamp collection increased by the same percent each year. However, each year, this percent was applied to the previous year's value. So, for example, the increase from year 1 to year 2 is 6% of \$2,650, not 6% of the original \$2,500. This type of change is called **compound growth.**

In the next problem, you will continue to explore compound growth. You will consider the effects of both the initial value and the growth factor on the value of an investment.

Connecting Growth Factor and Growth Rate



Cassie's grandmother started college funds for her two granddaughters. She gave \$1,250 to Cassie and \$2,500 to Cassie's older sister, Kaylie. Each fund was invested in a 10-year bond that pays 4% interest a year.

A. For each fund, write an equation to show the relationship between the number of years, *n*, and the amount of money in the fund, *m*.

Cassie: ____________________ Kaylie: _____________________

b. Make a table to show the amount in each fund for 0 to 10 years.

Number of Years	Amount of Money in Funds	
	Cassie's Account	Kaylie's Account

C. 1. How does the initial value of the fund affect the yearly increase in value? _________________________

____

2. How does the initial value affect the growth factor? ____

3. How does the initial value affect the final value? ____

____

D. A year later, Cassie's grandmother started a fund for Cassie's younger brother, Matt. Cassie made this calculation to predict the value of Matt's fund several years from now:

Value = $2000 * 1.05 * 1.05 * 1.05 * 1.05

1. What is the initial value? ___________ growth factor? __________

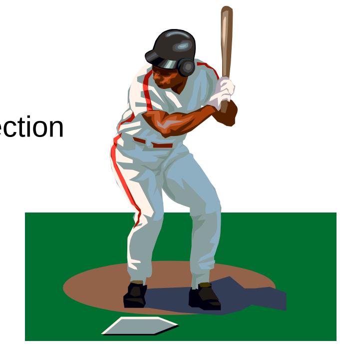
growth rate? __________ Number of years that Cassie is assuming? ___________

2. Write an equation to predict the value, *v*, of Matt's fund given *n* number of years. _______________

3. If the value continues to increase at this rate, how much would the fund be worth in 10 years?

(Show your work.) ____

<u>On Your Own:</u>



1. Tanner made the following calculations to predict the value of his baseball card collection several years from now:

value = $130 * 1.07 * 1.07 * 1.07 * 1.07 * 1.07

a. What is the initial value? ___________ growth factor? __________

growth rate? __________ Number of years that Tanner is assuming? ___________

b. Write an equation to predict the value, *v*, of Tanner's cards given *n* number of years. _____________

3. If the value continues to increase at this rate, how much would the cards be worth in 10 years?

(Show your work.) ____

2. **Multiple Choice** Ms. Diaz wants to invest $500 in a savings bond. At which bank would her investment grow the most over 8 years?

F. Bank 1: 7% interest for 8 years.

G. Bank 2: 2% interest for the first 4 years and 12% interest for the next four years.

H. Bank 3: 12% interest for the first 4 years and 2% interest for the next four years.

J. All three result in the same growth.

Part 3) Scientific Notation

Write in standard form.

1) 1.91×10^{-5} ________________ 2) 3.1×10^{7} ________________ 3) 8.46×10^{8} ________________

4) 9.182×10^{5} ________________ 5) 7.2×10^{-8} ________________ 6) 1.97×10^{-6} __________________

Write each number in scientific notation.

7) 0.000 000 027 ____________ 8) 79,000,000,000______________ 9) 0.000 000 398 ______________

10) 123,100,000______________ 11) 0.000 000 05 ___________ 12) 0.000 000 018______________

Use your calculator to evaluate the following. Write the answer in scientific notation and standard form. **Round to three significant digits.**

	Scientific notation	Standard form
13) 7^{-11}	________________________	________________________
14) 8^{12}	________________________	________________________
15) 5^{-15}	________________________	________________________
16) 6^{-10}	________________________	________________________
17) 9^{11}	________________________	________________________
18) 12^{-8}	________________________	________________________

Part 4: Each table shows exponential growth. Complete each table. Fill in the blanks.

a)

t	0	1	2	3	4
m	6	7.8	10.14	13.18	17.14

Growth factor:_____________ Original #: _____________

Equation: _____________ Percent of change (growth rate):_____________

b)

t	0	1	2	3	4
p	15	28.5	54.15	102.89	195.48

Growth factor:_____________ Original #: _____________

Equation: _____________ Percent of change (growth rate):_____________

c)

n	0	1	2	3	4	10
a	10	14	19.6			

Growth factor:______________ Original #: ______________

Equation: ______________ Percent of change (growth rate):______________

d)

s	0	1	2	3	4	20
m		11	11.22	11.4444	11.673	

Growth factor:______________ Original #: ______________

Equation: ______________ Percent of change (growth rate):______________

e)

t	0	1	2	3	4
m					

Growth factor:______________ Original #: ______8_________

Equation: ______________ Percent of change (growth rate):______10%_______

f)

t	0	1	2	3	4
p					

Growth factor:______1.5_________ Original #: ______12_________

Equation: ______________ Percent of change (growth rate):______________

g)

n	0	1	2	3	4	10
a						

Growth factor:______1.05_________ Original #: ______200_________

Equation: ______________ Percent of change (growth rate):______________

h)

t	0	1	2	3	4
p					

Growth factor:______________ Original #: ______40_________

Equation: ______________ Percent of change (growth rate):______2.5%_______

i)

n	0	1	2	3	4	10
a						

Growth factor:______________ Original #: ______1_________

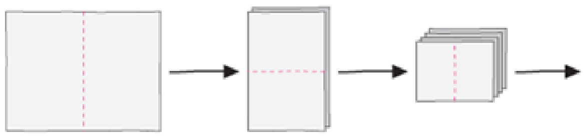
Equation: ______________ Percent of change (growth rate):______150%________

Growing, Growing, Growing Investigation 4

Exponential Decay…Introduction

1) Size of the Ballots

In Problem 1.1, you read about ballots Chen, the secretary of the SGA, is making for a meeting. Recall that Chen cuts a sheet of paper in half, stacks the two pieces and cuts them in half, stacks the resulting four pieces and cuts then in half, and so on.



You investigated the pattern in the number of ballots created by each cut. In this problem, you will look at the pattern in the areas of the ballots.

A. The paper Chen starts with has an area of 64 square inches. Complete the table to show the area of a ballot after each of the first 10 cuts.

#of Cuts	Area (in^2)
0	64
1	32
2	
3	
4	
5	
6	
7	
8	
9	
10	

B. How does the area of a ballot change with each cut?

____

C. Write an equation for the area, *A*, of a ballot after any cut, *n*.

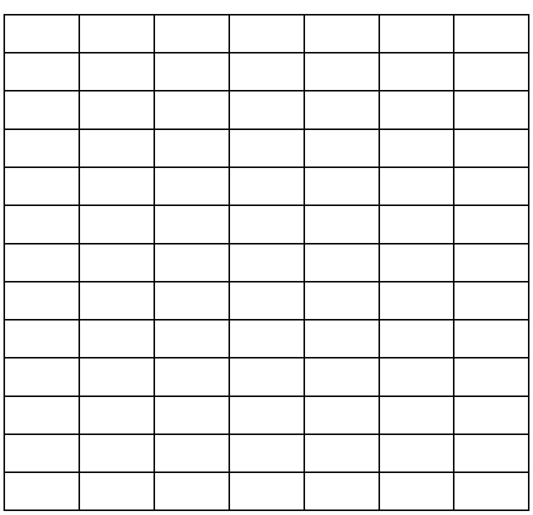
____

D. How is the pattern of change in the area different from the exponential growth patterns you studied? How is it similar?

____

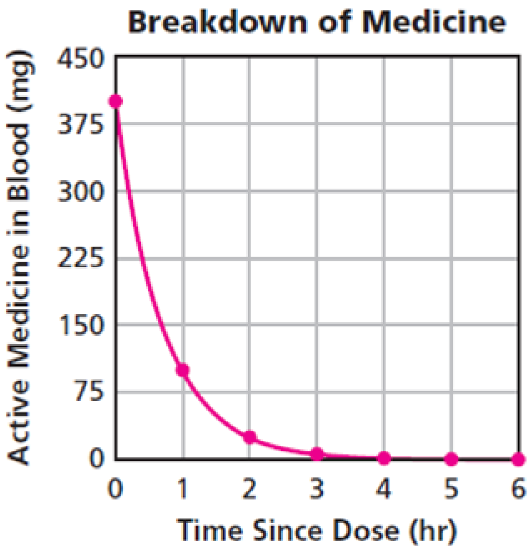
____

E. Make a graph of the first 7 cuts. Use an interval of 1 on the x-axis and 5 on the y-axis.



Exponential patterns like this, in which a quantity decreases at each stage, show **exponential decay**. The factor the quantity is multiplied by at each stage is called a **decay factor**. A decay factor is always less than 1, but greater than zero. In this ballot problem, the decay factor is $\frac{1}{2}$.

2) Fighting Fleas
After an animal receives a preventive flea medicine, the medicine breaks down in the animal's bloodstream. With each hour, there is less medicine in the blood. The table and graph show the amount of medicine in a dog's bloodstream each hour for 6 hours after receiving a 400-milligram dose.



Breakdown of Medicine

Time Since Dose (hr)	Active Medicine in Blood (mg)
0	400
1	100
2	25
3	6.25
4	1.5625
5	0.3907
6	0.0977

Study the pattern of change in the graph and the table.

1. How does the amount of active medicine in the dog's blood change from one hour to the next?

____

2. What is the decay factor? (Show your work.) What is the original amount?

______________________________ ______________

3. Write an equation to model the relationship between the number of hours, *h*, since the dose is given and the milligrams of active medicine, *m*.

____

4. How is the graph for this problem similar to the graph you made in the "Ballot Area" problem?

____

3) Tables to Equations Complete each table. State the decay factor, the original # (y-intercept), and write the equation.

A)

x	0	1	2	3	4
y	100	60	36	21.6	

Decay factor: __________ Original #: __________ Equation: ____________________

B)

x	0	1	2	3	4	10
y	250	50	10			

Decay factor: __________ Original #: __________ Equation: ____________________

C)

x	0	1	2	3	4	10
y	40	8	1.6			

Decay factor: __________ Original #: __________ Equation: ____________________

On your own....

1) Andrea has a 24-inch string of licorice to share with her friends. As each friend asks her for a piece, Andrea gives him or her half of what she has left. She doesn't eat any of the licorice herself.

a. Make a table and a graph showing the length of licorice Andrea has left each time she gives a piece away. (Use an interval of 1 on the x-axis and 2 on the y-axis.)

# of Friends	Length of Licorice
0	24
1	
2	
3	
4	
5	
6	

b. Suppose that, instead of half the licorice that is left each time, Andrea gives each friend 4 inches of licorice. Make a table and a graph showing the length of licorice Andrea has let each time she gives a piece away. (Use an interval of 1 on the x-axis and 2 on the y-axis.)

# of Friends	Length of Licorice
0	24
1	
2	
3	
4	
5	
6	

c. Compare the tables and the graphs for the two situations. Explain the similarities and differences.

____

____

2. Penicillin decays exponentially in the human body. Suppose you receive a 300-mg dose of penicillin to combat strep throat. About 180 mg will remain active in your blood after 1 day.

a. Assume the amount of penicillin active in your blood decreases exponentially. Make a table showing the amount of active penicillin in your blood for 7 days after a 300 mg dose.

Time Since Dose(days)	Active Medicine in Blood (mg)
0	
1	
2	
3	
4	
5	
6	
7	

b. Write an equation for the relationship between the number of days, d, since you took the penicillin and the amount of medicine, m, remaining active in your blood.

____

c. What would be the equation if you had taken a 400-mg dose?

____

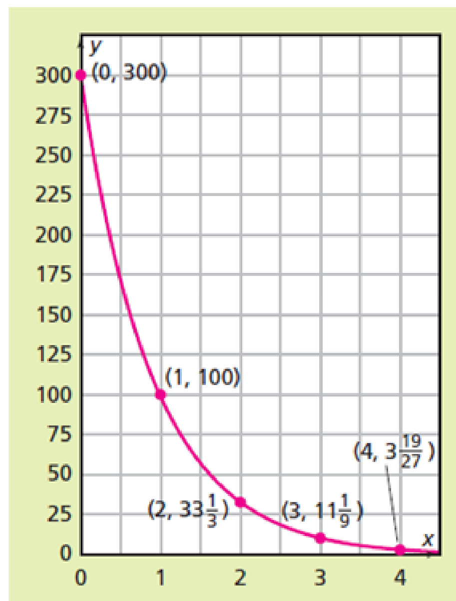
3. The graph toward the right shows an exponential decay relationship.

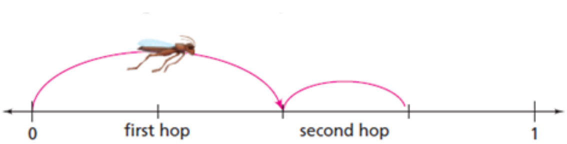
a. What is the decay factor? _________

b. What is the y-intercept? __________

c. Write the equation for the graph.

____





4. A cricket is on the 0 point of a number line, hopping toward 1. She covers half the distance form her current location to 1 with each hop. So, she will be at $\frac{1}{2}$ after one hop, $\frac{3}{4}$ after two hops, and so on.

a. Make a table showing the cricket's location for the first 10 hops.

Hop	0	1	2	3	4	5	6	7	8	9	10
Location											

b. Where will the cricket be after *n* hops? __________________________

c. Will the cricket ever get to 1? Explain. ____

____

5. **Tables to Equations** Complete each exponential table. State the decay factor or growth factor, the original # (y-intercept), and write the equation.

A)

x	0	1	2	3	4
y	90	9	0.9	0.09	

Decay or Growth Factor: _______ Original #: ________ Equation: ____________________

B)

x	0	1	2	3	**10**
y		120	72	43.2	

Decay or Growth Factor: _______ Original #: ________ Equation: ____________________

C)

x	0	1	2	3	**10**
y		39	101.4	263.64	

Decay or Growth Factor: _______ Original #: ________ Equation: ____________________

D)

x	0	1	2	3	**10**
y		59.5	505.75	4298.875	

Decay or Growth Factor: _______ Original #: ________ Equation: ____________________

E)

x	0	1	2	3	**10**
y		24	9.6	3.84	

Decay or Growth Factor: _______ Original #: ________ Equation: ____________________

F)

x	0	1	2	3	**10**
y	0.6	1.5	3.75		

Decay or Growth Factor: _______ Original #: ________ Equation: ____________________

G)

x	0	1	2	3	**10**
y	0.5	0.3	0.18		

Decay or Growth Factor: _______ Original #: ________ Equation: ____________________